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Properties and a constitutive model for frozen and unfrozen soil

Ronald Brinkgreve, Delft University of Technology / Plaxis

Authors: Manuel Aukenthaler, Ronald Brinkgreve, Adrien Haxaire

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Introduction

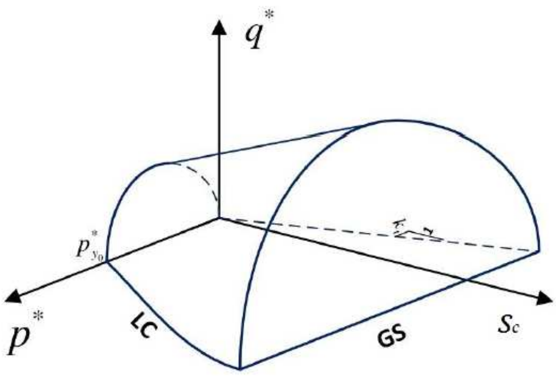
Frozen/unfrozen soils:

- Cold regions
- Climate change
- Artificial ground freezing

Introduction

Research on modelling of frozen/unfrozen soils i.c.w. NTNU

- Constitutive model (NTNU)
- Parameters and Validation (Plaxis / TUDelft)



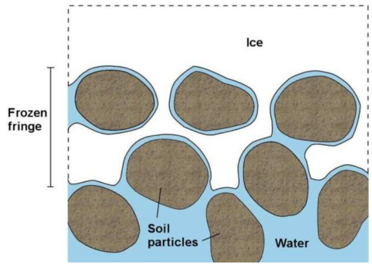
Characteristics of frozen soils

Transition pore water > pore ice (and vice versa):

- Gradual process (in t and T)
- Transition zone: Frozen fringe
- Unfrozen water content $\theta_{uw}(T)$
  (Soil Freezing Characteristic Curve, SFCC)
- Pressure melting $\theta_{uw}(p)$
- Change of hydraulic conductivity $k(T,p)$
- Change of stiffness and strength
- Expansion / compression

Part 1

Part 2



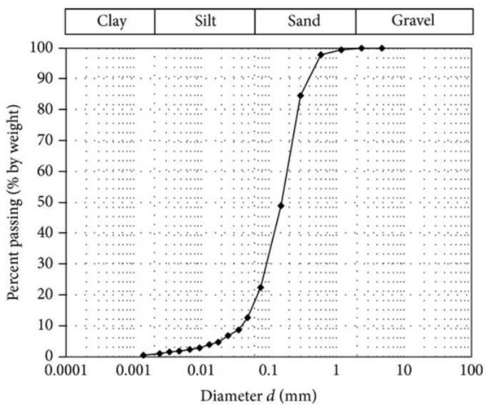
Characteristics of frozen soils

Problem:

- Temperature-related parameters are uncommon for geotechnical engineers

Solution:

- Derive them from common soil data
  (Particle Size Distribution)



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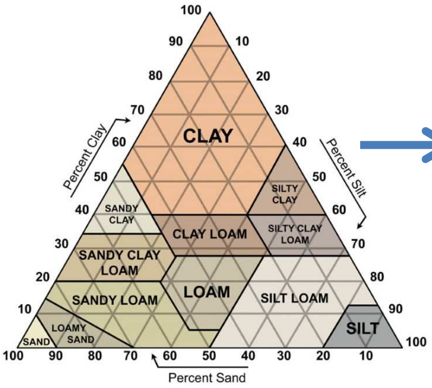
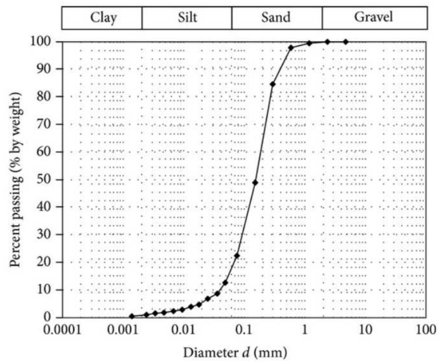
Starting from particle size distribution…

Selection of soil type > PSD > SSA





Specific Surface Area (SSA)

$$SSA = 3.89 \cdot d_g^{-0.905}\ [\mathrm{m}^2/\mathrm{g}]$$

$$d_g = \exp(m_{cl} \ln d_{cl} + m_{si} \ln d_{si} + m_{sa} \ln d_{sa})\ [\mathrm{mm}]$$

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Soil-Freezing Characteristic Curve

SSA > Unfrozen water content θ_{uw}: (after Anderson & Tice, 1972)

$$\theta_{uw} = \frac{\rho_w}{\rho_b} \exp(0.2618 + 0.5519 \ln(SSA) - 1.4495 (SSA)^{-0.2640} \ln(T_f - T))$$

ρ_w = bulk density of water
ρ_b = bulk density of unfrozen soil
T = temperature
T_f = freezing temperature of water

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Soil-Freezing Characteristic Curve

Pressure-dependence of the freezing temperature:

$$\frac{p_{ice}}{\rho_{ice}} - \frac{p_w}{\rho_w} = -L \ln \frac{T}{T_f}$$

(Clausius-Clapeyron equation)

p_{ice} = pore ice pressure
p_w = pore water pressure
ρ_{ice} = bulk density of ice
L = latent heat

Cryogenic suction s_c:

$$s_c = p_{ice} - p_w$$
$$= \rho_{ice}\left(\frac{p_w}{\rho_w} - L \ln \frac{T}{T_f}\right) - p_w$$
$$\approx -\rho_{ice} L \ln \frac{T}{T_f} \qquad \textbf{(1)}$$

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Soil-Freezing Characteristic Curve

Melting pressure for ice p_{melt}: (after Wagner et al., 2011)

$$\frac{p_{melt}}{p_t} = 1 + \sum_{j=1}^{3} a_j \left(1 - \left(\frac{T}{T_t}\right)^{b_j}\right)$$

j	a_j [−]	b_j [−]
1	0.119539337×10^7	0.300000×10^1
2	0.808183159×10^5	0.257500×10^2
3	0.333826860×10^4	0.103750×10^3

$$p_{melt} = p_{ice} = s_c + p_w \qquad (T = T_f)$$

$T_t = 273.16\ \text{K}$ $\quad p_t = 611.657\ \text{Pa}$ (vapour-liquid-solid triple point)

$$\frac{s_c + p_w}{611.657\ \text{Pa}} = 1 + \sum_{j=1}^{3} a_j \left(1 - \left(\frac{T_f}{273.16\ \text{K}}\right)^{b_j}\right) \qquad \textbf{(2)}$$

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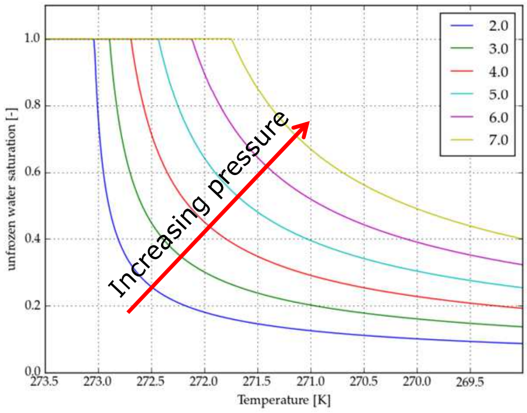
Soil-Freezing Characteristic Curve

Pressure melting:

- For a given p_w and T:
  s_c, p_{ice} and T_f can be calculated by solving eq. **(1)** and **(2)** iteratively

(pressure in MPa)



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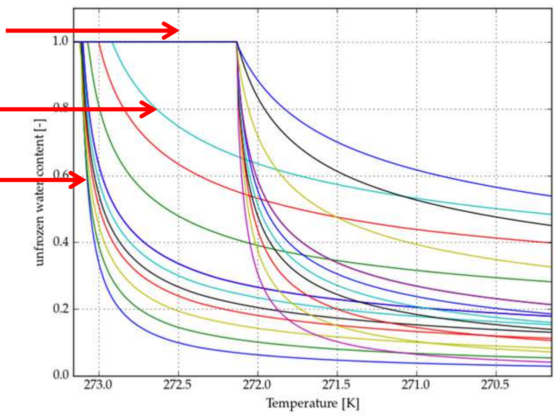
Soil-Freezing Characteristic Curve

Final result: SFCC for different soils at different pressures

Increasing pressure

Fine grained

Coarse grained



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Hydraulic conductivity

PSD > k_{sat} > $k(\theta_{uw})$: (after Tarnawski & Wagner, 1996)

$$k_{sat} = 4 \times 10^{-5}\left(\frac{0.5}{1-\theta_{sat}}\right)^{1.3b} \cdot \exp(-6.88 m_{cl} - 3.63 m_{si} - 0.025)\ [\mathrm{m/s}]$$

$$b = d_g^{-0.5} + 0.2\sigma_g\ [-] \qquad \sigma_g = \exp\left[\sum_{n=1}^{3} m_i (\ln d_i)^2 - \left(\sum_{n=1}^{3} m_i \ln d_i\right)^2\right]^{0.5}$$

Permeability of partially frozen soil:

$$k = k_{sat}\left(\frac{\theta_{uw}}{\theta_{sat}}\right)^{2b+3} = k_{sat}(S_{uw})^{2b+3}$$

m_i = mass fractions of clay, silt, sand
d_i = particle size limits
θ_{uw} = unfrozen water content
θ_{sat} = saturated water content

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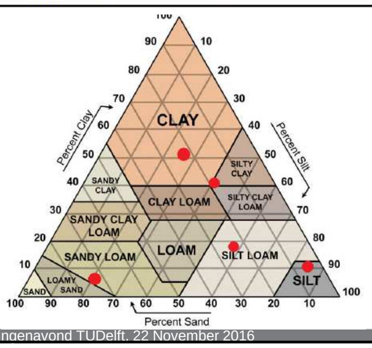
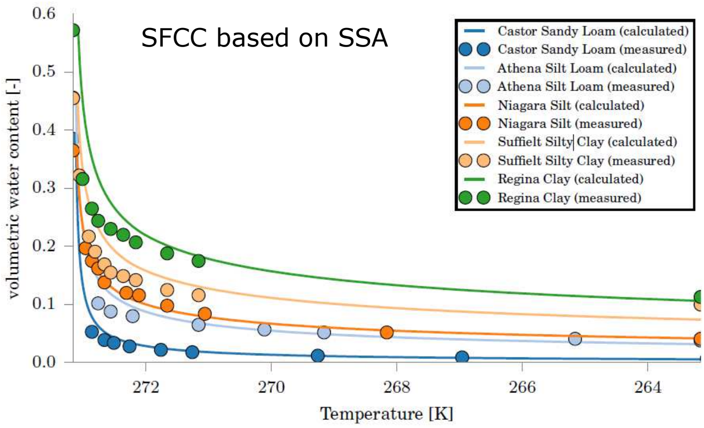
Validations

Soil	m_{clay}	m_{silt}	m_{sand}
Castor Sandy Loam	0.06	0.22	0.72
Athena Silt Loam	0.15	0.58	0.27
Niagara Silt	0.08	0.87	0.05
Suffielt Silty Clay	0.41	0.41	0.18
Regina Clay	0.52	0.25	0.23

(soil mass fractions after Smith & Tice, 1983)





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Validations

Pressure-dependence: (after Zhang et al., 1998)

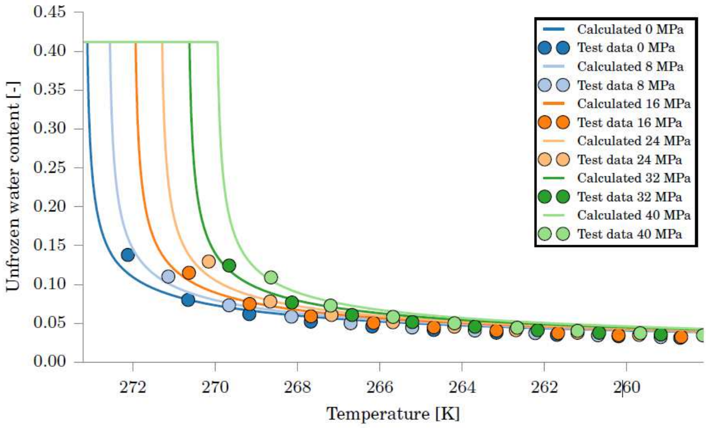
Lanzhou Loess:

m_{cl} = 0.12

m_{si} = 0.80

m_{sa} = 0.08

e_0 = 0.7



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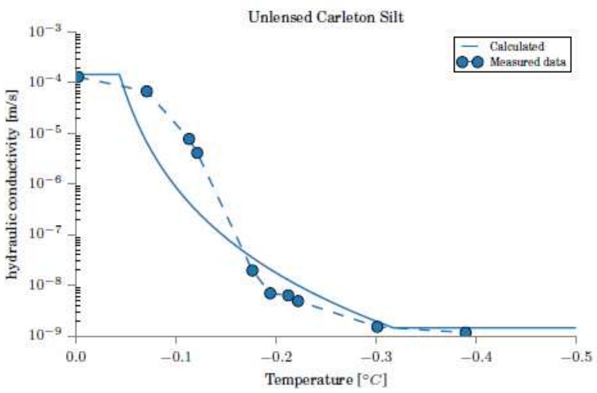
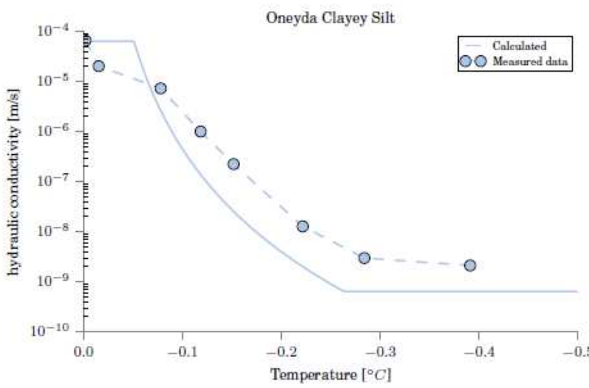
Validations

Hydraulic conductivity: (after Burt & Williams, 1976)





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Constitutive model for frozen/unfrozen soil

Solid state (grains + ice) stress σ*:

$$\boldsymbol{\sigma}^* = \boldsymbol{\sigma} - S_{uw} p_w \boldsymbol{I}$$

$$d\epsilon = d\epsilon^{me} + d\epsilon^{se} + d\epsilon^{mp} + d\epsilon^{sp}$$

σ = net stress
S_{uw} = unfrozen water saturation
p_w = pore water pressure
ε^m = solid phase strain component
ε^s = cryogenic suction strain component

Similarities with BBM
In unfrozen state: MCC

Constitutive model for frozen/unfrozen soil

Elastic strains:

$$d\epsilon_v^e = \frac{1}{K} dp^* + \frac{\kappa_s}{1+e} \frac{ds_c}{(s_c + p_{at})}$$

$$d\epsilon_q^e = \frac{1}{3G} dq^*$$

s_c = cryogenic suction

$$K = (1 - S_{ice}) \frac{(1+e) p_{y0}^*}{\kappa_0} + \frac{S_{ice} E_f}{3(1 - 2\nu_f)}$$

$$G = (1 - S_{ice}) G_0 + \frac{S_{ice} E_f}{2(1 + \nu_f)}$$

$S_{ice} = (1 - S_{uw})$
ice saturation

Frozen soil properties:

$$E_f = E_{f,ref} - E_{f,inc}(T - T_{ref})$$

Temperature-dependent (but stress-*in*dependent) Young's modulus of frozen soil

ν_f = frozen Poisson's ratio

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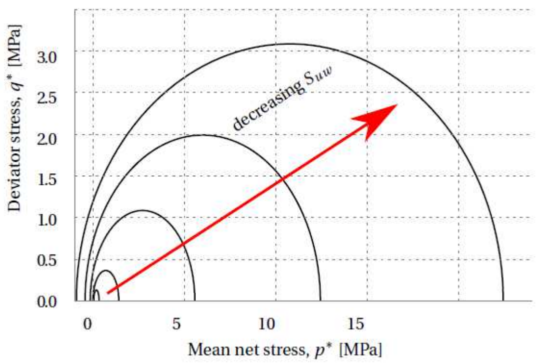
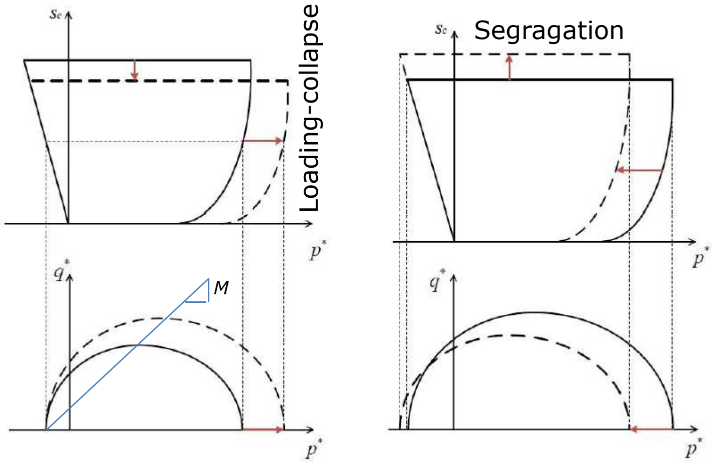
Constitutive model for frozen/unfrozen soil

Plastic strains:





(a) Plastic compression due to variation of solid phase stress (b) Plastic dilation due to occurrence of ice segregation

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Model parameters

Elastic parameters

Strength

Primary loading
- isotropic stress
- freezing

Parameter	Description	Unit
G_0	Unfrozen soil shear modulus	N/m^2
κ_0	Unfrozen soil elastic compressibility coefficient	–
$E_{f,ref}$	Frozen soil Young's modulus at a reference temperature	N/m^2
$E_{f,inc}$	Rate of change in Young's modulus with temperature	N/m^2/K
ν_f	Frozen soil Poisson's ratio	–
m	Yield parameter	–
γ	Plastic potential parameter	–
$(p^*_{y0})_{in}$	Initial pre-consolidation stress for unfrozen condition	N/m^2
p^*_c	Reference stress	N/m^2
λ_0	Elasto-plastic compressibility coefficient for unfrozen state	–
M	Slope of the critical state line	–
$(s_{c,seg})_{in}$	Segregation threshold	N/m^2
κ_s	Elastic compressibility coefficient for suction variation	–
λ_s	Elasto-plastic compressibility coefficient for suction variation	–
k_t	Rate of change in apparent cohesion with suction	–
r	Coefficient related to the maximum soil stiffness	–
β	Rate of change in soil stiffness with suction	(N/m^2)$^{-1}$

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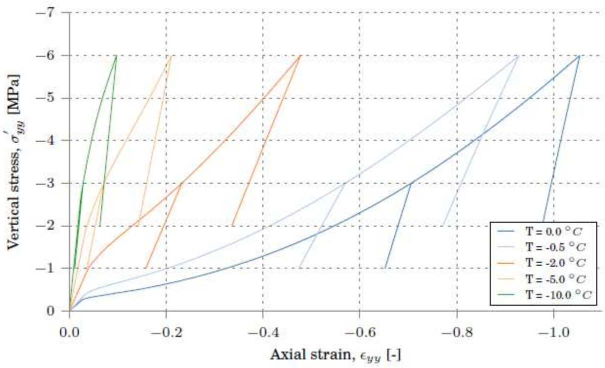
Model parameters

Parameter determination:

- From oedometer test (frozen/unfrozen):
  $(p_{y0}^*)_{in}$, p_c^*, β, κ_0, λ_0, r
- From simple shear test:
  G_0, M
- From axial compression test:
  $E_{f,ref}$, $E_{f,inc}$, ν_f, k_t
- From frost heave test:
  $(s_{c,seg})_{in}$, κ_s, λ_s



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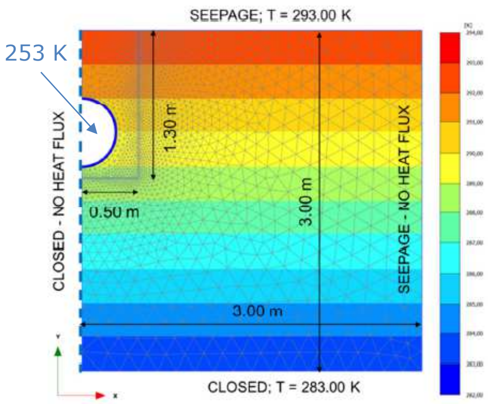
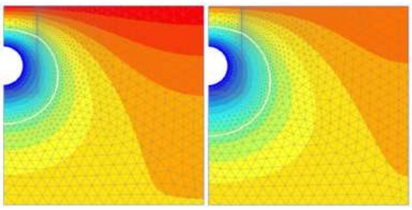
Applications – Frost heave

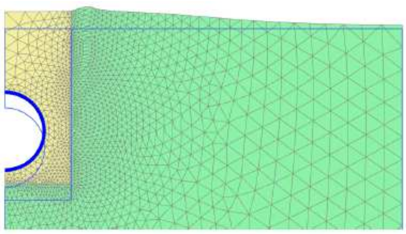
Chilled pipeline





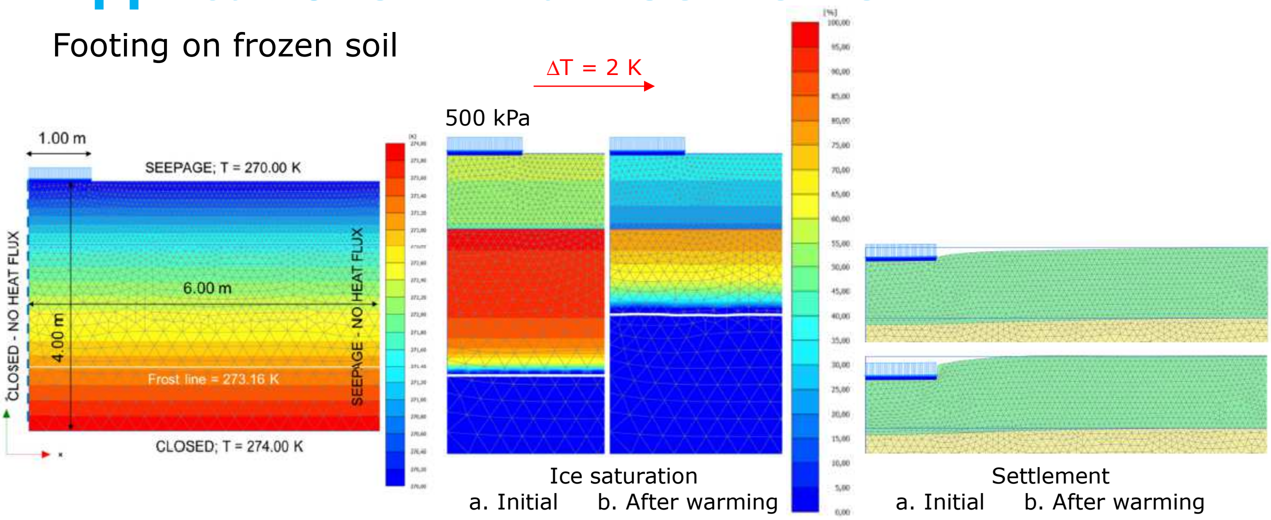
Temperature distribution
a. 40 days
b. Long time

Frost heave after long time



Applications – Thaw settlement

Footing on frozen soil



Conclusions

- Practical approach for $\theta_{uw}(T,p)$ and $k(T,p)$ of frozen soils
- Validated against data from literature
- PSD seems to enable a good estimate of properties of frozen soils
- Constitutive model for frozen / unfrozen soil
- Typical phenomena of frozen soil
- Successful applications:
  - Frost heave
  - Thaw settlement

Thank You