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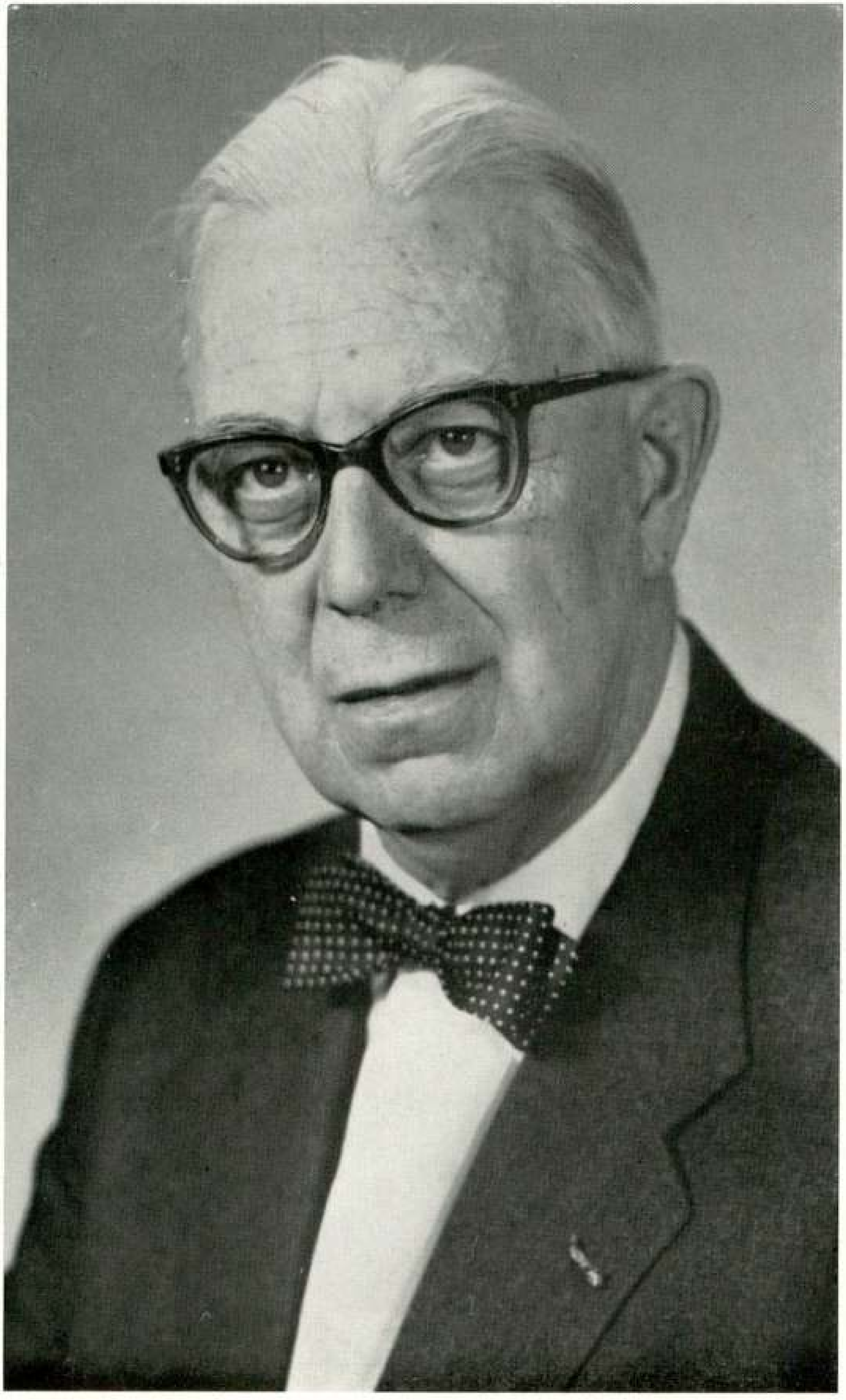

IN MEMORIAM PROF. DR. BALTH. VAN DER POL

In de nacht van 5 op 6 oktober overleed op zeventigjarige leeftijd ons erelid Prof. Dr. Balth. van der Pol. Hij was vele jaren voorzitter van het Nederlands Radiogenootschap en vormde tezamen met de heren Prof. Elias, Ir. Nordlohne, Dubois en Wesselius haar eerste bestuur. De oprichting in 1920 viel in het begin van een lange loopbaan, waarin Van der Pol een wereldnaam zou verwerven op het gebied van de radiowetenschap. Zijn proefschrift "De invloed van een geioniseerd gas op het voortschrijden van electromagnetische golven en toepassingen daarvan op het gebied der draadloze telegrafie en telefonie en bij metingen aan glimlichtontladingen" (Utrecht 1920) is een van de oudste verhandelingen waarin sprake is van het ons nu zo vertrouwde beeld van het ionosfeerplasma. Dit geschrift is een van de vroegste in een lange reeks van publicaties, die alle fundamentele kwesties betreffen op radiogebied in de meest ruime zin van het woord. Van de onderwerpen, die meer in het bijzonder zijn aandacht hadden noemen wij: antennetheorie, triode-oscillaties, frequentiemodulatie, de voortplanting van radiogolven, netwerktheorie, inschakelverschijnselen, de theorie van de grondslagen van de muziek en niet-lineaire trillingsverschijnselen. Het laatste onderwerp leidde tot uitvoerige studie van relaxatietrillingen; deze kunnen in veel gevallen beschreven worden door de naar hem genoemde "Vergelijking van Van der Pol". Het onderzoek van deze trillingen voerde voorts tot een theorie van de hartslag, die geillustreerd kon worden met behulp van een electrisch model (het kunsthart), waarvoor later grote belangstelling van medische zijde ontstond.

Wat de wiskundige behandeling betreft behoren de genoemde publicaties tot het terrein der toegepaste wiskunde. De bestudering ervan wekte vooral later zijn belangstelling op voor vele onderwerpen uit de zuivere wiskunde, in het bijzonder de getallentheorie en de theorie der Laplace transformaties (operatorenrekening). In dit laatste gebied bouwde hij voort op de door Heaviside geintroduceerde symbolische rekenwijze. Wellicht heeft geen ander zozeer als Van der Pol de betekenis van het niet altijd voldoende gewaardeerde werk van Heaviside doorzien.

Van der Pol's publicaties kenmerken zich door een heldere betoogtrant waarin de nadruk gelegd wordt op de algemene gedachtengang. Hij heeft voorts dikwijls de aandacht gevestigd op problemen die later door anderen op streng wiskundige wijze verder uitgewerkt werden. De grote betekenis van zijn werk werd door tal van buitenlandse onderscheidingen onderstreept, waarvan we in het bijzonder noemen de toekenning van de Medal of Honour door het Institute of Radio Engineers (in 1935) en van de Valdemar Poulsen Gold Medal door de Deense Academie van Technische Wetenschappen (in 1953). Sinds 1947 was hij lid van de Koninklijke Nederlandsche Academie van Wetenschappen.

Na beeindiging van zijn universitaire studie in Utrecht (van 1911 tot 1916) is Van der Pol zijn wetenschappelijke loopbaan begonnen met een verblijf in Engeland (1916-1919), alwaar hij achtereenvolgens studeerde onder Fleming in Londen en onder J. J. Thomson in Cambridge. In Nederland teruggekeerd werkte hij van 1919-1922 in Teyler's Stichting te Haarlem onder leiding van Lorentz, wat hijzelf steeds als een groot voorrecht heeft beschouwd. Het meeste van Van der Pol's scheppende arbeid is echter voortgekomen in de periode van 1922 tot 1949, waarin hij werkzaam was aan het Natuurkundig Laboratorium van de N.V. Philips te Eindhoven. Hier werd hij in 1925 hoofd van een groep medewerkers, die zich met de meest wetenschappelijke aspecten van het radio onderzoek bezig hielden; in verband hiermede werd hij in 1946 benoemd tot directeur voor fundamenteel radio-onderzoek.

Van der Pol was niet alleen een inspirerend leider voor research arbeid, doch had ook grote organisatorische gaven. Mede van hem stamt het initiatief tot oprichting van het Nederlands Radiogenootschap, terwijl hij van het begin af een belangrijke rol gespeeld heeft in de U.R.S.I. (Union Radio Scientifique International); van dit laatste instituut is hij lange tijd vice-president geweest, en sinds 1952 ere-president.

Enkele andere bijzondere gebeurtenissen in Nederland waren zijn benoeming tot bijzonder hoogleraar aan de Technische Hogeschool te Delft in 1938, zijn functie als voorzitter van de Tijdelijke Academie die in 1945 in Eindhoven gevestigd werd als voorbereiding voor het na de oorlog te herstellen technisch hoger onderwijs in Delft, en zijn benoeming tot Curator van het Mathematisch Centrum te Amsterdam·

Naast de zuiver wetenschappelijke hadden de technische en zelfs administratieve aspecten van het radiowezen zijn belangstelling. Hij was hierdoor bij uitstek de persoon om voor overheidsorganen als adviseur op te treden voor technische radio-problemen. Hij werd dan ook ondermeer lid van de 'Radioraad', terwijl hij een grote rol speelde bij het internationale instituut C.C.I.R. (Comité Consultatif International des Radiocommunications) dat adviserend moest optreden voor het opstellen van internationale regelingen van alle radioverkeer. In 1948 werd besloten tot het oprichten van een permanent Secretariaat van dit laatste instituut, ongeveer op het tijdstip waarop Van der Pol bij Philips de pensioengerechtigde leeftijd bereikte. Het was dan ook begrijpelijk dat hij wegens zijn grote theoretische kennis en zijn ervaring op technisch administratief gebied verzocht werd het Directeurschap van de C.C.I.R. te aanvaarden. Onder zijn leiding werden de werkzaamheden van dit Instituut òp een hoog peil gebracht en waakte hij ervoor, dat het wetenschappelijk element in het administratief diplomatieke werk van dit instituut niet te veel op de achtergrond geraakte. Einde 1956 moest hij wegens zijn gevorderde leeftijd aftreden als directeur van de C.C.I.R., doch van zijn verworven ervaringen werd hierna nog dankbaar gebruik gemaakt. Dit gold in het bijzonder het verkrijgen van internationale bescherming van de frequentiebanden, die van het grootste belang zijn voor het radio-astronomisch onderzoek.

Einde augustus moest hij zijn werkzaamheden ook op dit terrein, verricht in samenwerking met Prof. Oort, onderbreken wegens zijn verslechterende gezondheidstoestand. Dit was dus slechts kort voor het einde van zijn door zo grote activiteit gekenmerkte leven.

Van der Pol zal bij allen, die hem gekend hebben in de herinnering blijven voortleven als een warmvoelend mens, die zich ook sterk voor hun levensomstandigheden interesseerde.

Bij wetenschappelijke besprekingen was hij zeer geanimeerd en bekend om de grote geestigheid die hij aan de dag kon leggen.

In het bijzonder kwam hij steeds op voor de belangen van zijn medewerkers en maakte hij van zijn talrijke relaties gebruik om de aandacht te vestigen op hun prestaties. Na zijn vertrek naar Genève bleven de banden met velen van zijn vroegere Eindhovense medewerkers voortbestaan en men zag elkaar herhaaldelijk terug als oude vrienden.

Allen, die door omstandigheden een intensief contact met hem hebben gehad, hebben zeer veel aan hem te danken.

H. Bremmer

Fundamentals of colour television

by F. W. de Vrijer *)

Summary of a lecture read before the Nederlands Radiogenootschap on November 20th 1958.

This introductory lecture on colour television begins by recapitulating some colorimetric concepts, such as additive colour mixing and the chromaticity diagram (colour triangle). The principle of colour television is then explained with reference to a system containing three camera tubes and three picture tubes, considered at this stage with separate transmission channels for the three primary colours (red, green and blue). The splitting of the incident light into the three primary colours at the transmitting end, and the combination of the three (projected) pictures at the receiving end, can be effected by means of dichroic mirrors.

As regards transmission the main problem is to limit the bandwidth. A discussion follows of a system with two sub-carriers and of the N.T.S.C. system used in America. Finally, the author deals with the gamma correction needed in view of the non-linear relation between luminous flux and control voltage in picture tubes.

The problems dealt with in this article are discussed in detail in 'Fundamentals of colour television', Philips techn. Rev. 19, 86-97, 1957/58 (No. 3).

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Studio equipment for colour television

by A. G. van Doorn *)

Paper read before the Nederlands Radiogenootschap on November 20th 1958.

Summary

This paper gives a broad survey of the equipment constructed in the Philips Research Laboratories for the generation of colour-television signals and the testing of the different pick-up systems. It describes three colour cameras, one using Image Orthicons as pick-up tubes, while in the other two experimental Vidicons are used. Further the principle of the Flying Spot Scanner is described, as well as the Colour Film Camera. The special problems encountered in designing simultaneous pick-up systems and concerning colour-image registration, signal uniformity and gamma correction are discussed in more detail. In conclusion more is said about the different pick-up tubes and their use in colour-television cameras, their sensitivity, picture quality and overall performance.

1. Introduction.

In a colour-television studio much equipment is required for generating, processing and monitoring colour-television signals. First we need a generator for the synchronising and test signals. Then there are different signal sources, providing the video signals. A switching and distributing system is needed for the selection of signals at different locations and to switch the signals on to the actual line as the programme requires. Finally complete colour monitors are required at each signal source and at other points to permit of checks on the colour-picture quality.

The generator for the synchronising signals differs from generators for monochrome television because all synchronisation pulses must have a harmonic relationship to a standard frequency, viz. the colour-subcarrier frequency. As the tolerances for this standard frequency are very stringent (it must be accurate to 3 parts in a million), the oscillator should be extremely stable.

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The signal sources which translate optical pictures into electrical signals belong to three different categories according to their specific function. First: the live-pickup in the studio or outdoors is done by colour-television cameras. Secondly: the generation of electrical signals corresponding to colour transparencies is done by a flying-spot scanner. Finally: the generation of colour-television signals from motion-picture film is effected by colour-film scanners.

The basic principles of these different systems will be discussed in the three following paragraphs, and subsequently some paragraphs will be devoted to the most important problems which arise during practical operation.

2. **The colour camera.**

Up to now most colour cameras are built on the principle of simultaneous pick-up, by three pick-up tubes, of the three colour components of the scene. Thus the red, green and blue signals are provided simultaneously. As already described by de Vrijer [1]), the light coming from the scene has to be split into three components, each of which must fall on a separate pick-up tube which will translate the light component into an electrical signal. Thus a colour camera may be looked upon as a combination of three identical cameras directed at the same scene and provided with a light-splitting device.

2.1. *The light-splitting device.*

Figure 1 shows schematically a simple solution for such a

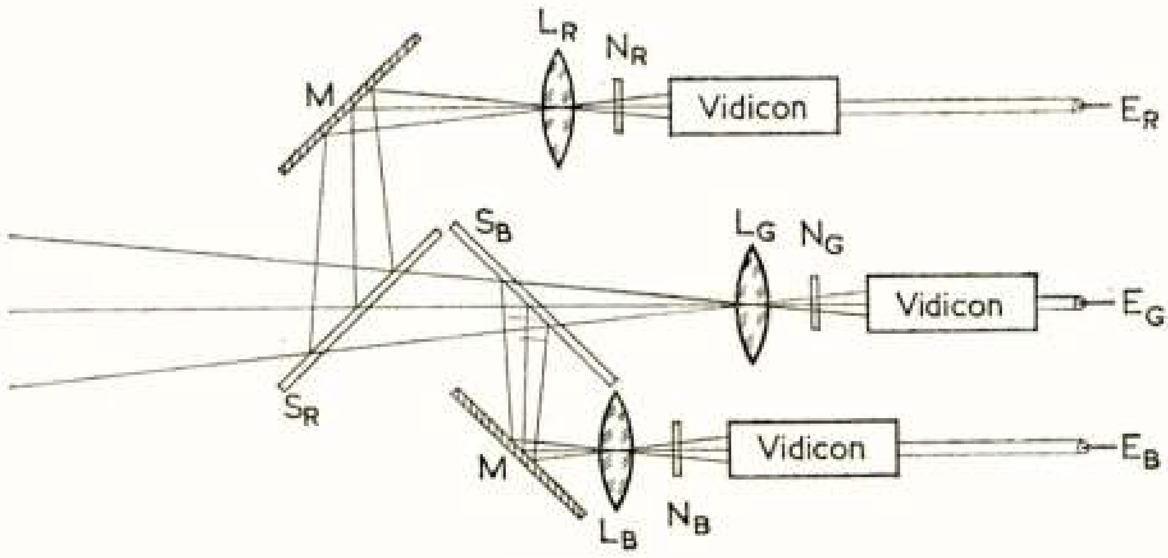

Fig. 1
Diagram of the light-splitting system in a colour-television camera with dichroic mirrors and separate objective lenses before each pick-up tube.

light-splitting device. The light from the scene falls on a dichroic mirror S_R, which reflects the red part of the light and transmits the green and blue parts. This transmitted light falls on a second dichroic mirror S_B, which reflects the blue component.

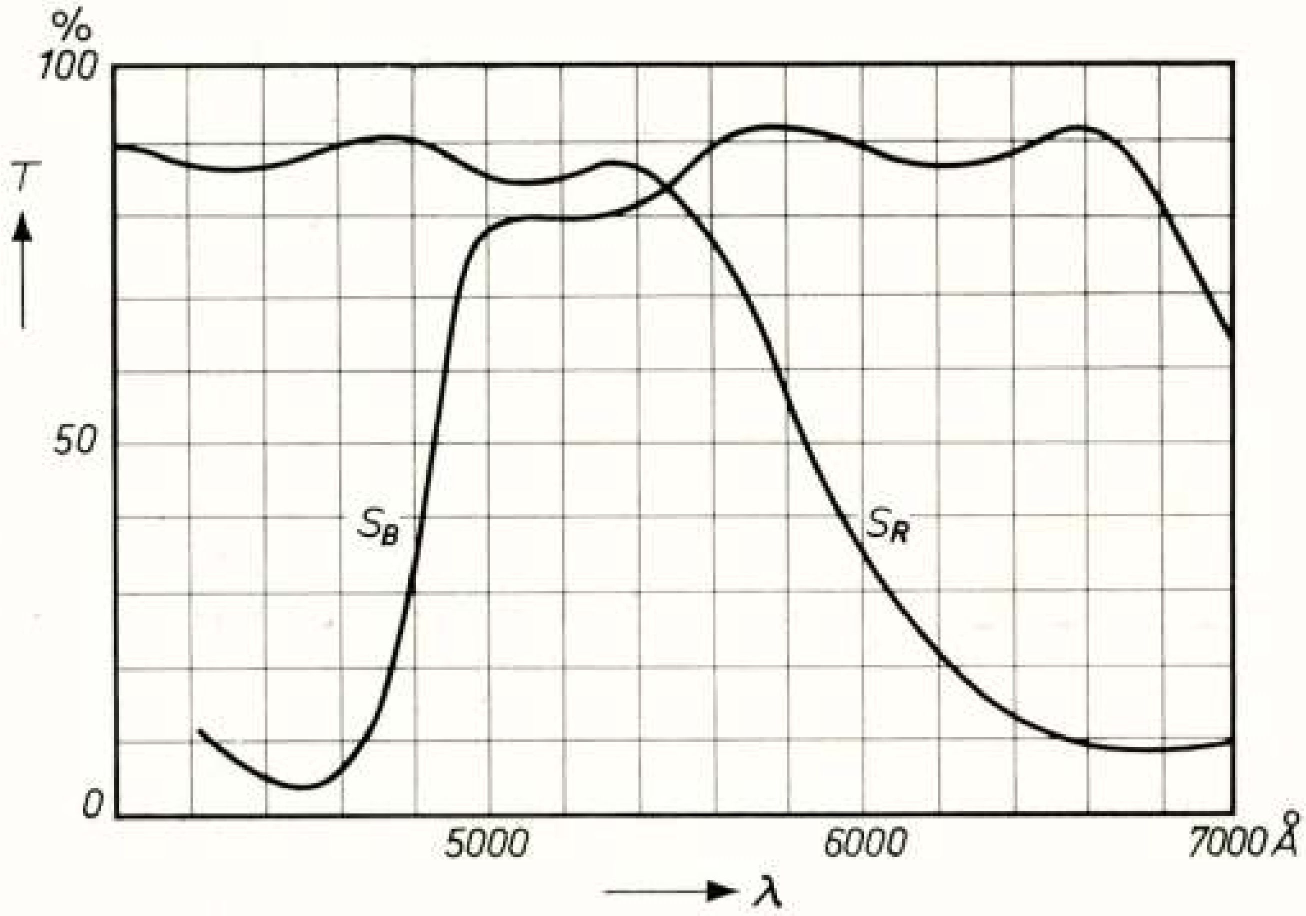

Fig. 2

Transmission characteristics for typical dichroic mirrors in use in colour cameras for splitting the light from the scene into three colour components.

Typical spectral transmission characteristics of these mirrors are given in fig. 2, which shows the transmission T as a function of wavelength. The dichroic mirrors are obtained by evaporating, onto glass, thin layers of a material with a high index of refraction, alternated by thin layers of a material with a low refractive index in such manner that interference effects give the required spectral transmission curves [2]). In this way light splitting can be obtained without light losses.

The red light which is reflected by S_R is received, via a plane front-surface mirror, by lens L_R. Thus the 'red image' of the scene becomes available at the red pick-up tube. That part of the light which was transmitted by S_R and reflected by S_B is passed on by a front-surface mirror to lens L_B, which then produces the 'blue image' of the scene on the blue pick-up tube. Finally the light which was transmitted by both dichroic mirrors, S_R and S_B, on reaching lens L_G produces the green component of the scene on the pick-up tube in the centre. This tube supplies the video signal E_G which carries the 'green' information only.

The colour filters N, as shown in the light paths give very accurately the required spectral sensitivity for each individual channel, which cannot be obtained by means of mirrors only.

The parallel arrangement shown in Fig. 1 enables the focussing for different object distances simply by mounting the three pick-up tubes on a sliding frame which can be moved axially with respect to the fixed lenses.

2.2. *Practical solution.*

The first colour camera built some years ago was based on this simple light-splitting device. The pick-up tube was an experimental vidicon described earlier in this journal [3]). Fig. 3 shows a photograph of this camera partly dismantled. The

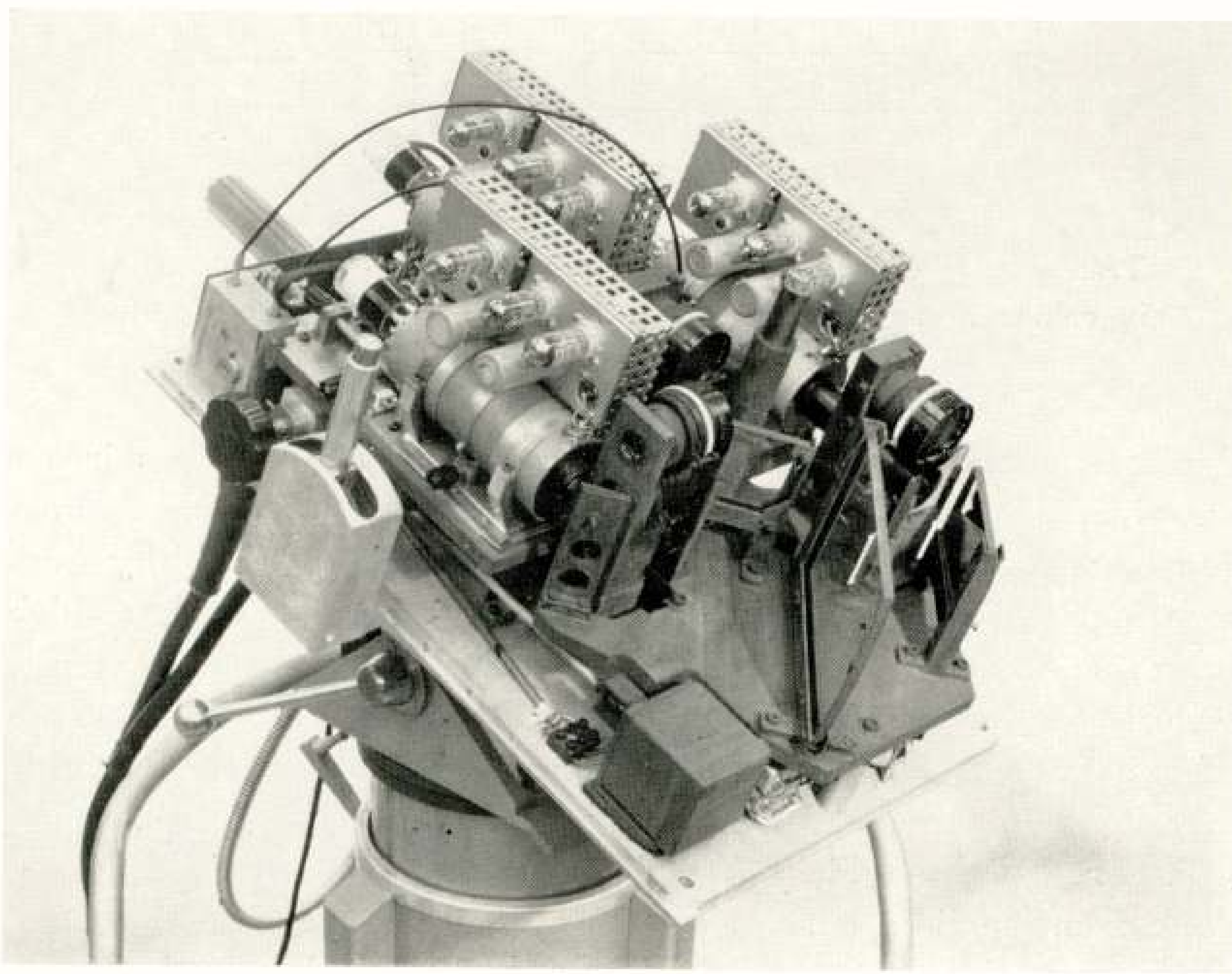

Fig. 3
Photograph of a dismantled colour camera to show the arrangement of the different components.

three parallel vidicons with associated camera pre-amplifiers, the lenses and the mirrors are plainly visible. To permit of the use of lenses with different focal lengths, involving the simultaneous replacement of all three lenses, a vertical movable lens holder is provided, carrying several sets of lenses. This system for changing lenses has the advantage of a high transmission factor, but there are important disadvantages too.

First, the choice of lenses is limited. Lenses with a short focal length require very large dichroic mirrors and the variation in angle of incidence of the light on the dichroic mirrors becomes too large to be ignored.

The demands to be made upon the mechanical design and the extensive optical adjustments are very high, as the slightest mutual differences give rise to registration errors of the individual colour-images when lenses are changed.

Third, the diaphragm adjustment must be effective for all lenses in the same way. These disadvantages and the sensitivity to dust and stray light make this optical system, although very efficient, less suitable for use in an all-purpose studio camera.

2.3. *Relay-lens system.*

The optical system which makes use of a relay lens and is shown in fig. 4 is much better suited to our purposes. When this system is applied, the camera can be equipped with a conventional lens turret as used for monochrome cameras. This lens turret contains lenses of different focal length. One of these lenses, e.g. L_1, produces a real image within the plane of a field lens, V_1. This image is reproduced on the photosensitive layer of the pick-up tube via a second optical system. Such an optical system may consist for example of 2 identical

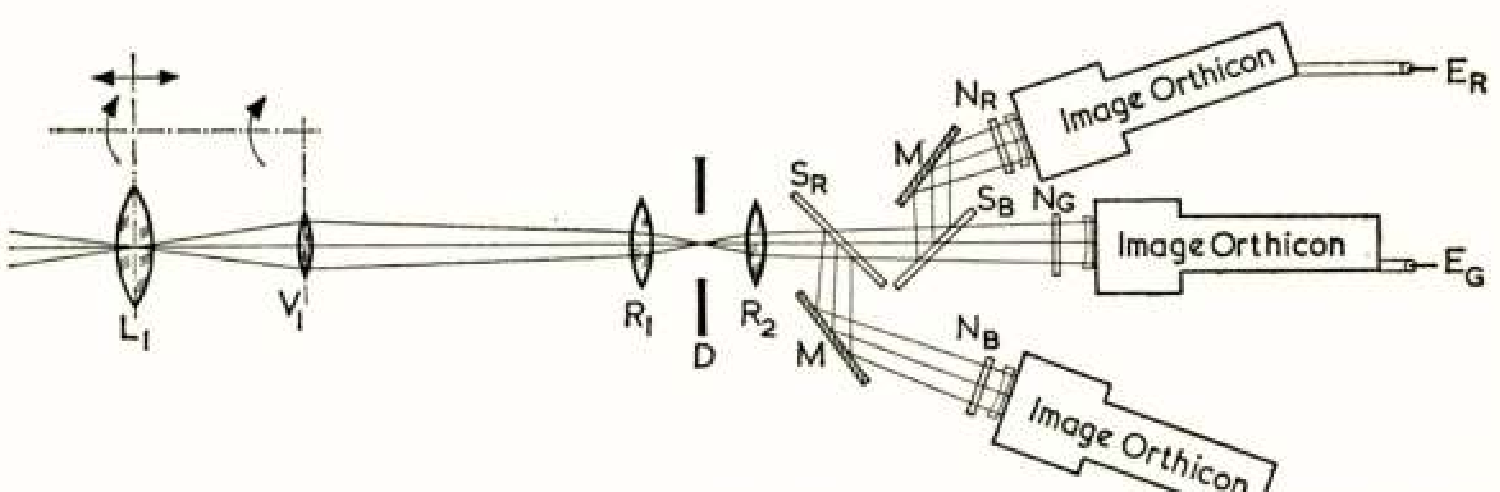

Fig. 4

Diagram of the relay optical system in an image-orthicon colour camera.

objective lenses, R_1 and R_2, with long focal lengths. The resulting free space must be sufficiently large to house the light-splitting mirrors and filters, which divide the light into three components as described above.

The astigmatism caused by the oblique plane-parallel plates in the convergent beam is eliminated by giving the plates a slight amount of prismaticy as described by de Lang [4]).

Correction lenses are placed in front of each pick-up tube to correct for the aberrations introduced by the field-lenses.

Several advantages of this system can be mentioned.

a. Lenses of any desired focal length may be selected by turning the lens turret so that the desired objective lens and the associated field lens are brought into working position. This lens replacement takes place in the common optical path, which ensures that no optical registration errors can occur between the colour images. Consequently the requirements imposed on the mechanical design are less stringent.
b. Focussing can be effected by axial displacement of the turret in such a way that lens L moves relative to field lens V. As a consequence there is no need for parallel arrangement of the pick-up tubes and the camera can be made more compact.
c. The light control which is needed on each camera is in this case of the single type and is effected in the second optical system (diaphragm D of one of the two objectives, R_1 or R_2).
d. The optical system can be built into a light- and dust-proof housing.

A great disadvantage of this method is the poor transmission, which is only half that obtained with the simple system described in section 2.2. If, however, there is no other alternative, the relay-lens system must be accepted and the only thing to do is to keep the reflection at the many glass surfaces low by a proper coating of the optical elements, which also improves the contrast.

2.4. *Image-orthicon camera with relay-lens system.*

The relay-lens system has been adopted for the camera shown in fig. 5, in combination with image orthicons as pick-up tubes. The size of these tubes with their focussing and deflection coils governs the dimensions of the other elements, including the optical ones.

The photograph shows the lens turret at the front. It has 4 lenses with focal lengths ranging from 5 cm to 15 cm and is turned by means of a handle at the rear of the camera. After the selection of another lens the focus is adjusted automatically, the mechanical arrangement being such that the turret shifts axially to the position required for optimum focus. The open cover makes one of the pick-up tubes visible, with its coil system and the associated video pre-amplifier. The closed

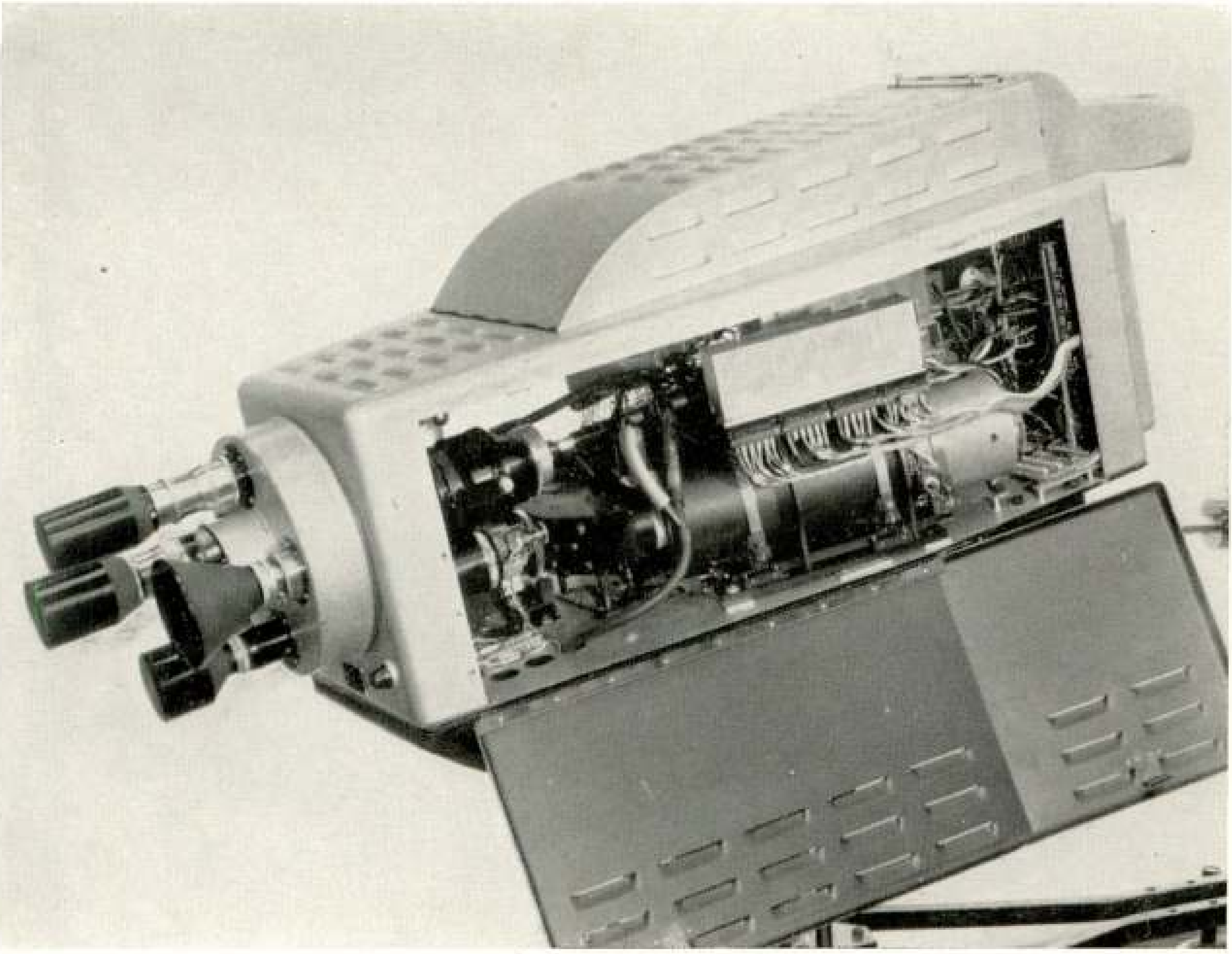

Fig. 5
Experimental image-orthicon colour camera with sidedoors open to show the arrangement.

box contains all the optical elements. The hoses are for forced-air cooling, which is necessary to keep the operating temperature of the image orthicons within certain limits in order to obtain the best picture quality. On top of the camera we see the electronic viewer. With the exception of the horizontal-deflection circuit, the blanking amplifier and the three pre-amplifiers all circuits are built in the control racks so as to avoid unnecessary heat dissipation inside the camera.

2.5. *Vidicon camera with relay lens.*

Fig. 6 shows a camera incorporating a perfected relay-lens system and vidicon pick-up tubes. The relay-lens system has a reduction factor of 2. Owing to this the optical system can have a large aperture with no significant optical errors, irrespective of the focal length. The efforts to find a compact arrangement have led to a non-symmetrical set-up with two tubes perpendicular to one another on the base plate and the third one on a higher level, partly on the optical box. Lens selection

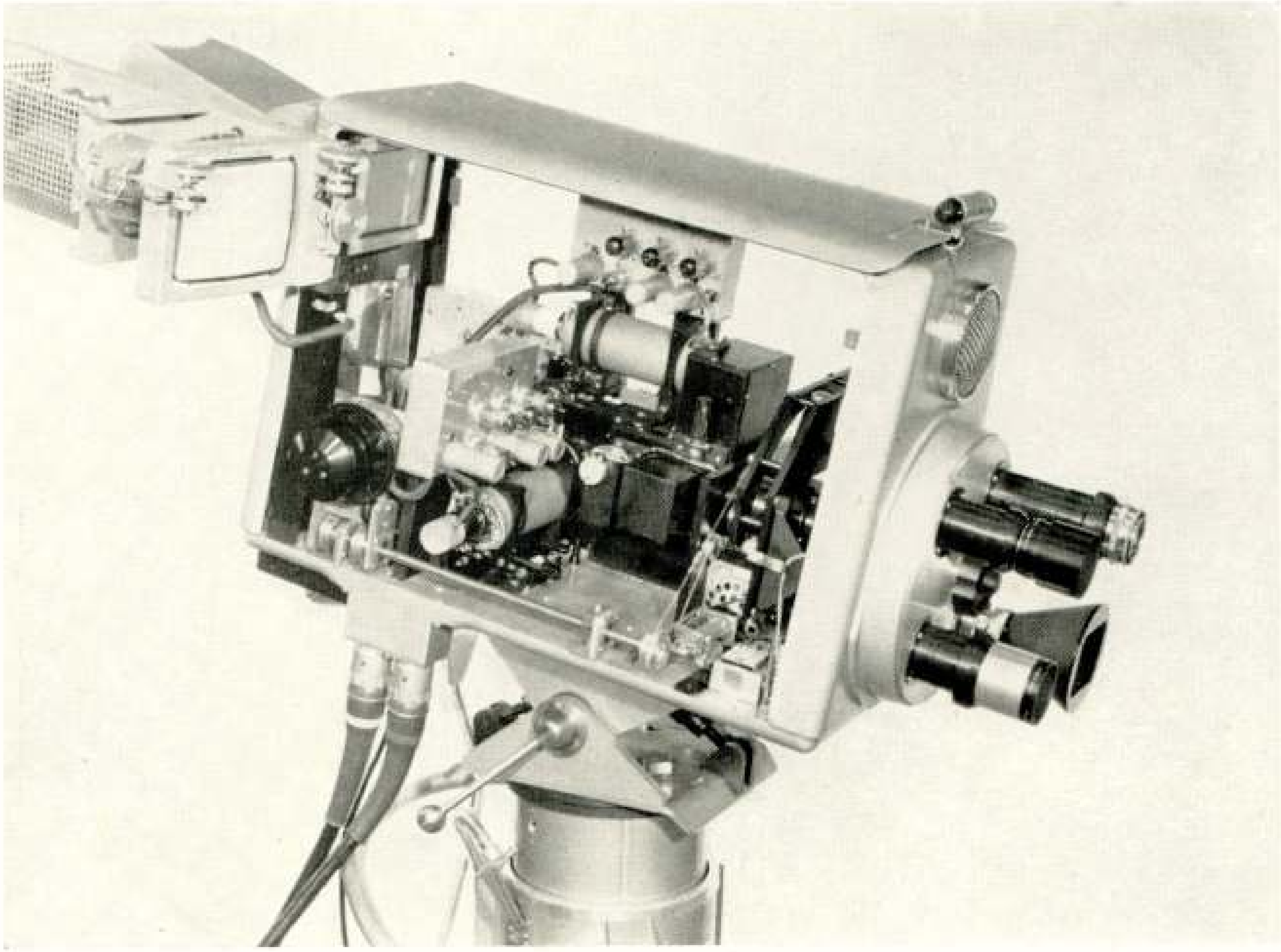

Fig. 6

Complete vidicon colour-television camera. The photograph shows the lens turret with four lenses, the arrangement of the optical system and the pick-up tubes, the different controls and the pivoted electronic viewfinder.

and focussing is done by means of a twin knob at the right-hand side of the camera. The lens replacement is carried out electrically, i.e. by a motor which rotates the lens turret via a Maltese cross. When the lens is changed the focus is adjusted automatically, the turret being shifted axially by a mechanical device. As the photograph shows the viewer is pivoted. This facilitates servicing of the camera.

3. **Flying-spot scanner.**

An important signal source which gives high-quality colour pictures is the flying-spot scanner. Its operating principles can be explained with reference to the schematic drawing of Fig. 7. The primary light source is a cathode ray tube with a phosphor having a short afterglow time. On this tube an ordinary television raster of high and uniform brilliance is scanned. The raster picture is thrown by lens L on to the colour transparency D. One could also say that the colour slide is scanned by a flying spot, moving in the same way as a scanning beam

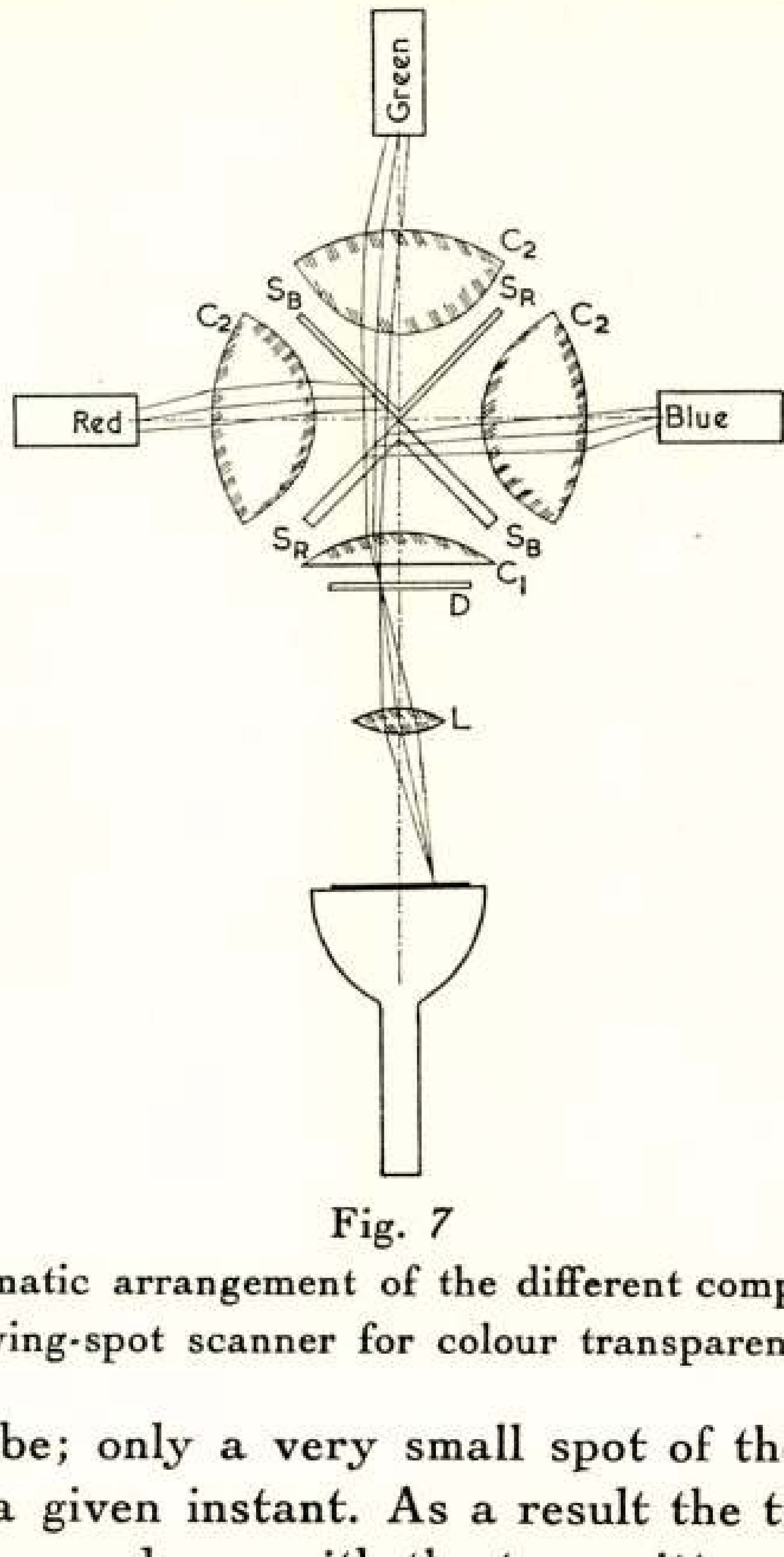

Fig. 7
Diagrammatic arrangement of the different components in a flying-spot scanner for colour transparencies.

in a pick-up tube; only a very small spot of the colour slide is illuminated at a given instant. As a result the transmitted light is modulated in accordance with the transmittance of the colour transparency. The light is collected by a condenser lens C_1 and split up into the three colour components by the dichroic mirrors, S_R and S_B. After splitting, each of the three colour components is passed through a second aspheric condenser lens, C_2, and made to strike a photomultiplier (this is a photocell with secondary-emission amplification). Here each component is converted into an electrical signal, the amplitude of which is proportional to the amount of light transmitted by the colour transparency for the primary colour in question. The high quality of the pictures, obtained with the flying-spot scanner, is due to several causes. In the first place there are no registration errors because a single raster is reproduced on the colour transparency by one single objective lens system so that the three colour images are geometrically identical. Moreover, uniform sensitivity over the picture area is obtained by imaging the exit pupil of the lens L on the photocathodes of the photomultipliers and by placing the dichroic mirrors in a telecentric part of the imaging beam.

4. Colour-film scanners.

To obtain electrical signals corresponding to motion-picture films, either of the two principles explained above could be used.

When the flying-spot scanner is used no problems will be encountered in connection with registration and the signals will be uniform over the picture area. However, the flying-spot scanner is a non-storage device and for this reason the mechanical design is rather complicated. The film images must each remain stationary during the whole scanning period and a very fast pull-down of the film becomes necessary. Or, should the film be moved continuously at constant speed, optical compensations will be necessary. Both designs require a high degree of mechanical and optical precision.

Alternatively a simple colour camera can be used for the generation of electrical signals from motion-picture film, in which case full use can be made of the storage properties of the pick-up tubes. A conventional film projector can then be used for projecting the images by means of a lens with a long focal length, via a dichroic-mirror assembly, onto three vidicon pick-up tubes. Fig. 8 shows a schematic drawing of the equipment used in the Philips Research Laboratories with good results.

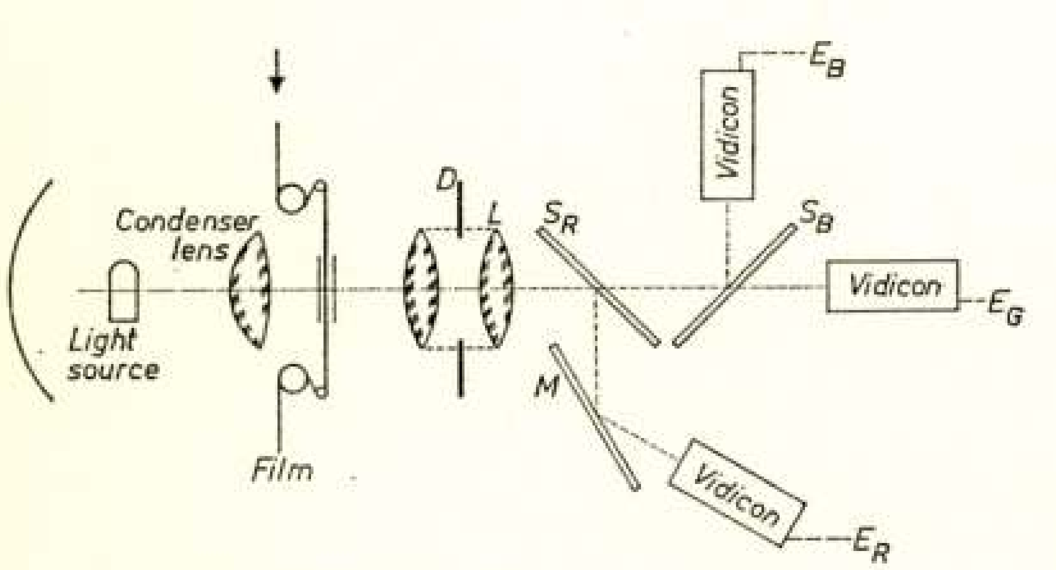

Fig. 8
Diagrammatic arrangement of an experimental vidicon colour-film camera.

5. Registration of colour images.

One of the great problems encountered with colour cameras with simultaneous pick-up is the necessity of obtaining three images which are geometrically identical. When the images are not identical we get a loss of definition and colour errors in the complete colour image. The three images must cover each other exactly if lack of sharpness and colour-fringing, are to be avoided.

As we have seen, the scene to be televised is displayed three times in a colour camera, one for each colour, on three pick-up tubes. The design of the optical system must be such that the three optical images are geometrically identical. In each pick-up tube the optical image is converted into an electrical image which is scanned by an electron beam, giving the video signal for one colour. In this paper we shall not discuss in detail the operation of the pick-up tubes. However, we must always be mindful of the fact that a pick-up tube, which can never have an ideally symmetrical set-up, is placed inside a deflection and focussing system, which in practice will always have small errors. So it is obvious that some geometric distortion will arise. And if this distortion is different for each of the three pick-up systems, then the registration of the colour images will be faulty. A geometrical difference between two colour images of, e.g. $\frac{1}{2}$ % means a registration fault of about 3 television lines, and a difference of 0.1 % may already give a noticeable reduction in definition. Hence great care must be taken to avoid such distortions. The optical system must be very accurately aligned and the focussing and deflection coils must be as identical as possible. But nevertheless it is necessary to introduce electrical corrections for small tolerance errors.

Fig. 9 shows a simplified schematic diagram of a deflection circuit for a vidicon camera in which provision has been made for several electrical adjustments. The three horizontal-deflection coils are fed in parallel from one single output transformer. In our equipment the deflection circuits are fitted in the control racks and the deflection coils are supplied via the camera cable. It is very important for this cable to be properly terminated at the camera end because transient reflections may give very disturbing ringing effects. For this reason the three branches with the horizontal-deflection coils are included in a network of parallel branches RL and RC with equal R. This network has a constant impedance if $RC = \dfrac{L}{R}$, forming in that case a termination resistance R for the cable. In a practical case R must be 75 ohms, so the total resistance in each branch should be 225 ohms. Each branch includes a variable inductance and a variable resistance, for linearity and amplitude control respectively.

A centring control is needed to adjust the relative position of the rasters. Therefore a variable direct current can be sent through the deflection coil to shift the whole raster horizontally. Furthermore it has often been found necessary to introduce a correction against the skew of the raster which is caused by the deflection coils for the two directions not being situated exactly at right angles. This skew error can be eliminated by introducing into the horizontal-deflection circuit a small vertical-sawtooth current which should be adjustable in amplitude and polarity.

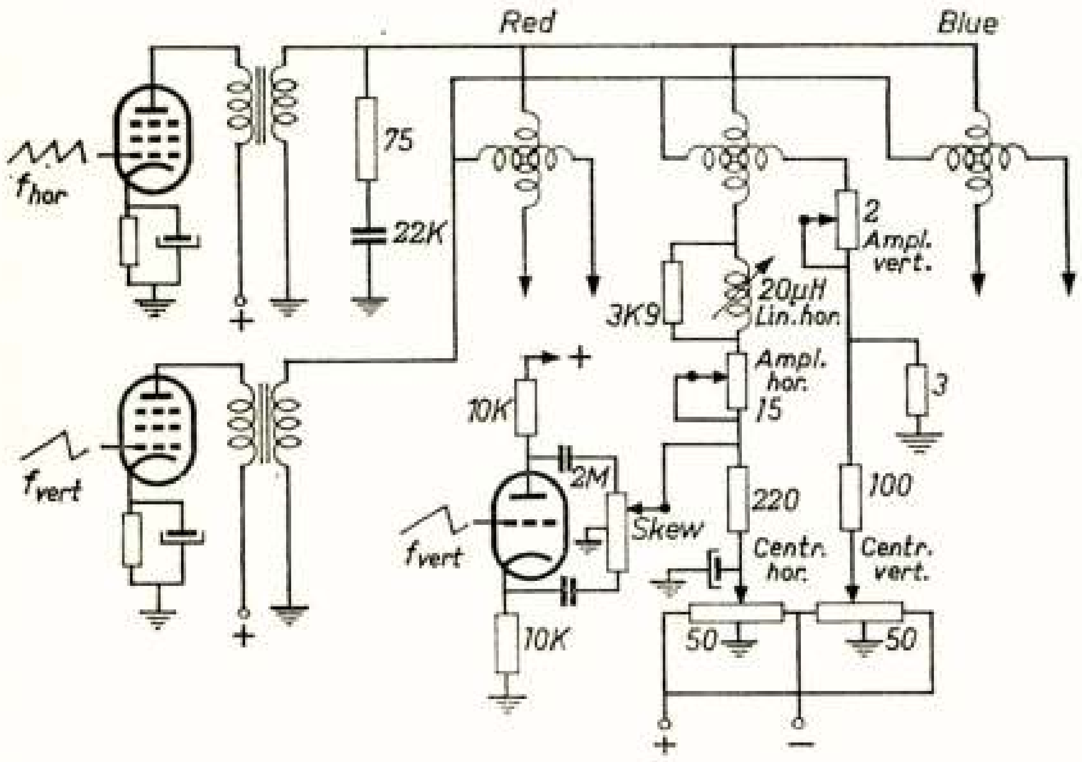

Fig. 9

Simplified schematic diagram of the deflection circuits in a vidicon colour camera with separate adjustments for amplitude and linearity, as well as centring controls and skew correction.

The arrangement of the vertical-deflection circuit is also drawn in fig. 9, showing the individual controls for vertical amplitude and centring.

In addition to these electrical adjustments a number of mechanical ones are desirable. Hence each deflection system, including the tube, is made movable, axially as well as transversely, relative to the optical image, while a slight rotation of the deflection system and the tube is also possible.

These measures make it easier to minimise registration errors. Perfect registration is very hard to obtain especially in the case of an image-orthicon colour camera. This pick-up tube is rather complicated and its adjustments are many; further its position in the focussing and deflection field is very critical. All this may easily give rise to small geometric distortions. Careful alignment, however, gives favourable results.

In the case of the vidicon camera these difficulties are less pronounced and with the available controls it will nearly always be possible to obtain good registration.

6. **Picture quality.**

The overall picture quality is not only influenced by the registration problem, but also by many other factors. The definition must, of course, be satisfactory and a good signal-noise ratio is essential. But the colour reproduction must also be good over the whole of the picture area and at all levels of brilliance.

6.1. *Signal uniformity.*

In practice almost all pick-up tubes show some degree of non-uniform sensitivity over the picture area. This may have several causes, which we shall ignore because of their complexity and because a discussion of the properties of pick-up tubes is outside the scope of the present paper.

The non-uniformity can take various shapes. The pick-up tube may produce a video signal which is not constant, even though it is illuminated uniformly. The tube sensitivity may be found to vary in a certain manner when we go from the centre of the photo-sensitive layer towards the edges, or from left to right, or from top to bottom, and vice versa. This non-uniformity may be different for the three pick-up tubes and in that case the colour reproduction will deteriorate. Even when there is no illumination, a pick-up tube produces a shading signal which is not uniform over the picture area. This tube property may give rise to colour differences in dark parts of the displayed picture and thus have a very disturbing effect. For this reason it is necessary that the pick-up tubes be selected carefully, so that their differences in these respects are as small as possible. Provision must also be made for introducing electrical corrections: white and blacklevel adjustments and a shading correction in the video amplifier. The shading correction may be effected by means of a correction signal which is added to the video signal and consists of sawtooth- or parabola-shaped voltages with line and frame frequency. Amplitude and polarity of this correction signal must be adjustable.

6.2. *Gamma correction.*

The transfer characteristic of a colour reproducing system has to be in principle linear, as non-linearity results not only in gray-scale distortion, but also in deterioration of colour reproduction [5]). However, the picture tube in a television receiver is not linear at all, the relationship between output luminance B and input voltage E being subjected to a power law: $B = E^{\gamma}$, where the exponent, γ, is about 2.5.

Therefore, in order to produce overall linearity (i.e. gamma equal to unity), another non-linear element is required. For reasons of simplicity and cost, this gamma correction is always introduced in the studio equipment before transmission.

We must also take into account the transfer-characteristic of the pick-up tube, which can also be non-linear, the power exponent varying between 0.7 and unity, depending on the type of pick-up tube, the selected tube setting and the mean signal content.

For good colour reproduction to be obtained it is important that the values of γ for red, green and blue should be capable of being made equal. To this end each video amplifier should be provided with an adjustable gamma corrector.

Fig. 10 shows the circuit diagram for a simple variable gamma corrector. The video signal is clamped at the grid of a tube which has in its cathode circuit a germanium diode in series with a resistance. The voltage-current characteristic of a germanium diode is a power-law characteristic. So, if the input is linear, the signal voltage across the diode is non-linear with a γ of about 0.4, depending on the series resistance and the d.c. bias of the diode.

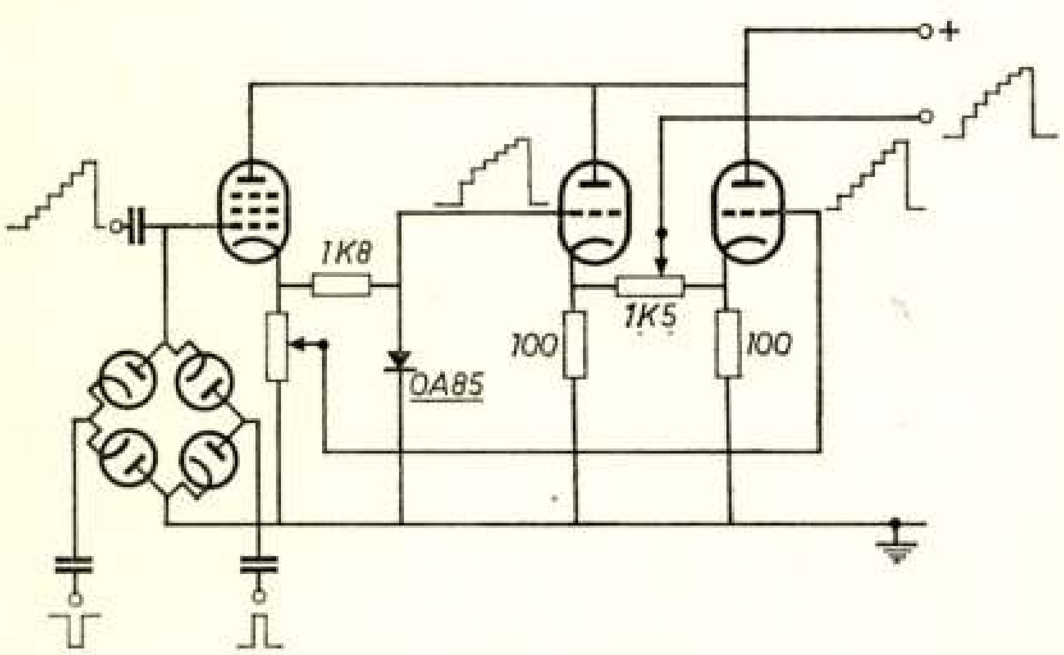

Fig. 10

Simplified schematic diagram of a gamma-correction circuit with adjustable gamma at constant amplitude.

Addition of part of the linear signal to the non-linear output of the diode may give a gamma variation between 0.4 and 0.9, approximately. In the circuit shown in Fig. 10 this is done with the aid of a potentiometer between the two cathodes which makes it possible to keep the total signal amplitude constant when the gamma of the signal is varied.

6.3. *Control equipment.*

The preceding discussion has made it clear that a rather large number of adjustments are required for obtaining a good picture quality.

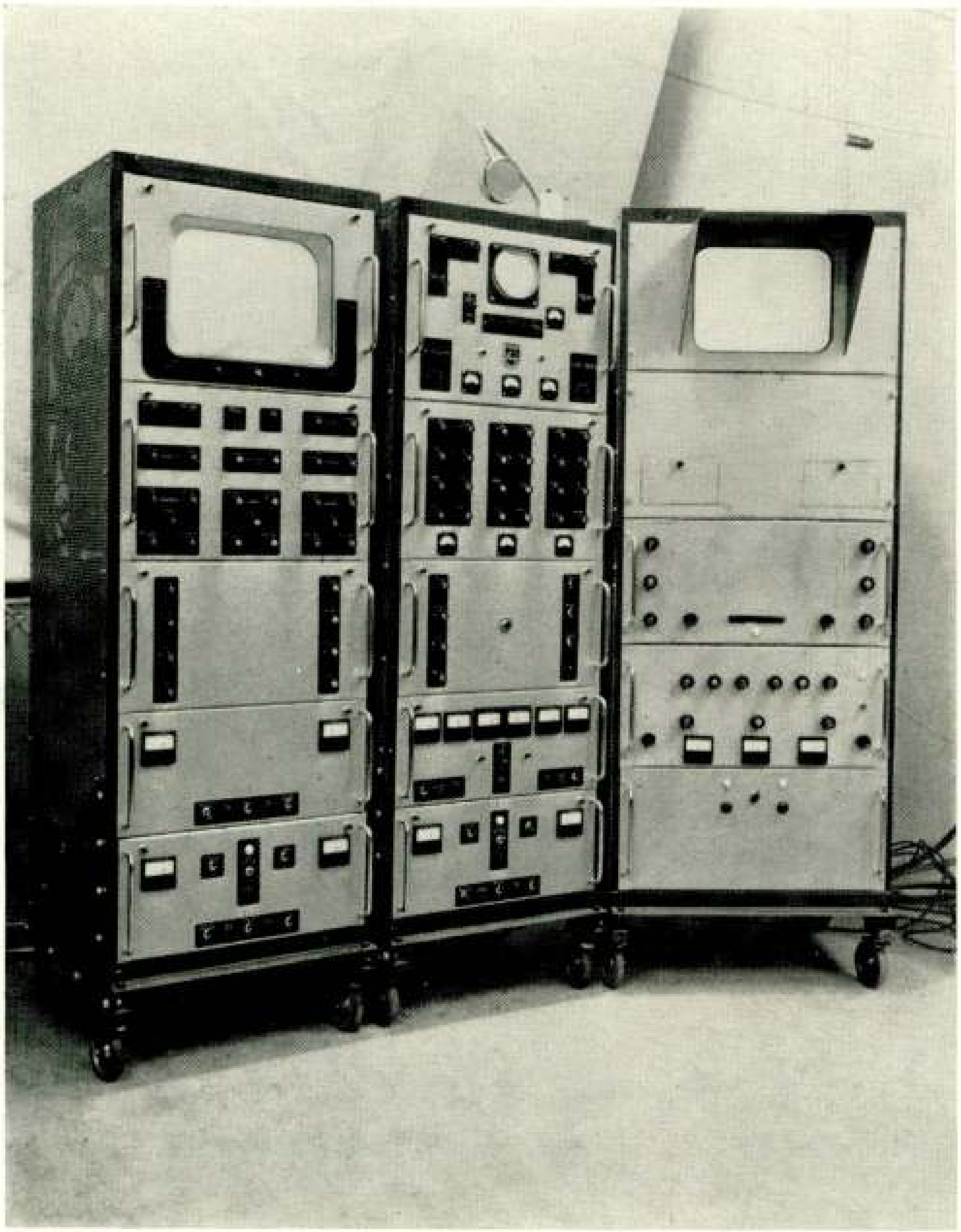

Fig. 11
Photograph of the control equipment of an image-orthicon colour camera. Right, a colour monitor of the projection type.

Fig. 11 shows the control equipment used for one single camera chain (in this case the image-orthicon chain). At the top left is a monitor which can be used to control the proper registration of the three colour images. Below that we see a panel carrying the adjustments for registration and shading correction. At the top centre is a monitoring panel with an oscilloscope, a temperature indicator and remote diaphragm control. Below that a panel carrying three sets of identical controls for the three image orthicons. In two other panels are housed the three video amplifiers with amplitude controls, black-level adjustments and variable gamma correctors. The rack to the right contains a colour monitor of the projection type [6]).

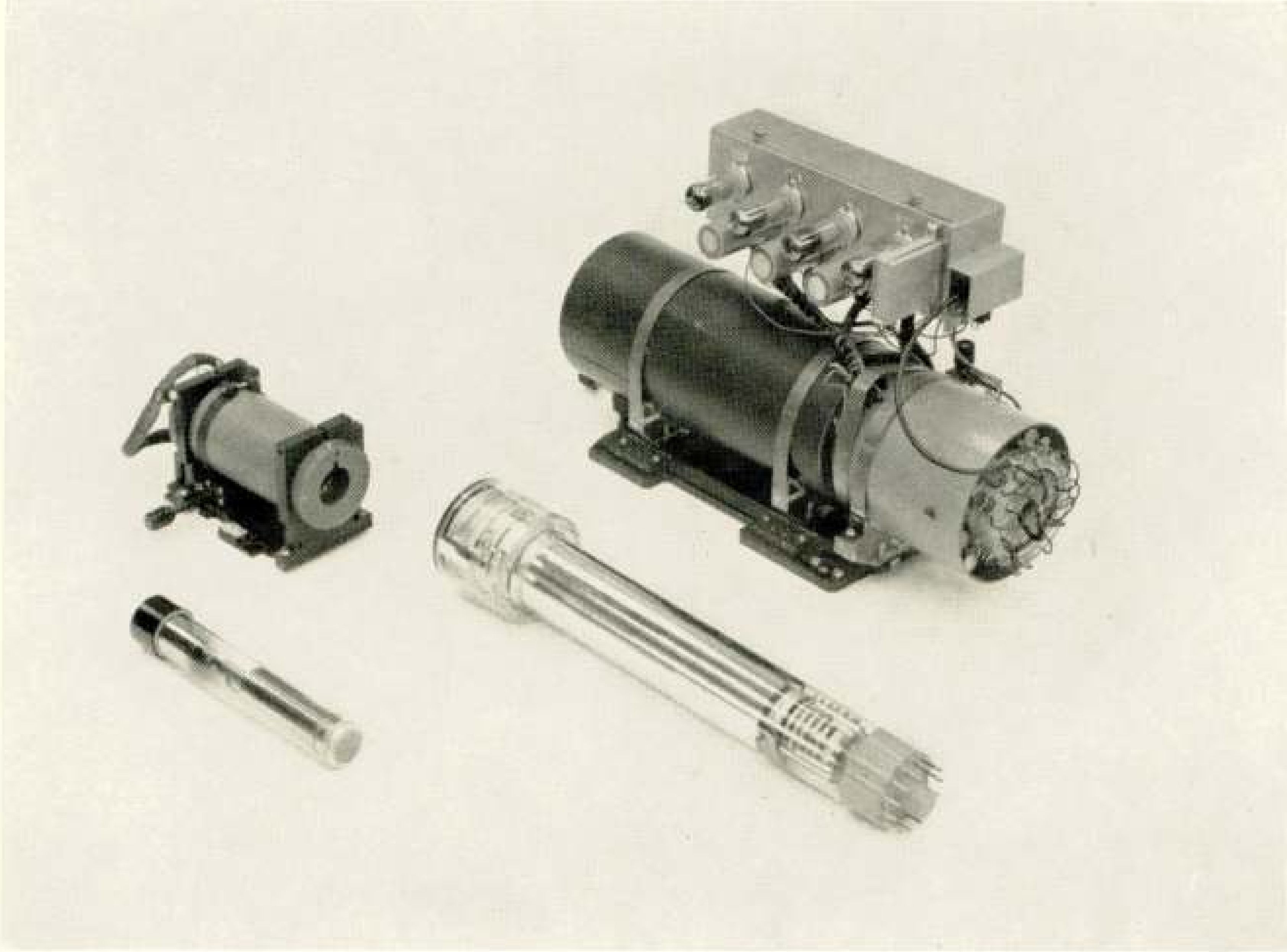

Fig. 12
Photograph of an image-orthicon pick-up tube (right) with its chassis carrying deflection and focussing system and pre-amplifier, and a vidicon with its complete chassis (left), showing the large difference in size.

7. Camera pick-up tube.

From the various types of pick-up tubes in use for monochrome television only two have been selected for colour television, viz. the image orthicon and the vidicon. For the televising of life scenes the former is the only commercially available tube that has sufficient sensitivity to operate at an acceptable low light level. This is an image orthicon especially developed for colour television. However, in cases where high light levels are available, as in film scanners and for surgical and medical purposes, the vidicon has great advantages.

We have had the opportunity of using in our experimental cameras a vidicon which is still in a laboratory development stage. This tube has special qualities for colour television. It may be used at low light levels without showing unacceptable trailing effects due to persistence or lag of the photoconductive layer. It has a very good black-level over the whole of the picture area as there is little or no dark current, so dark-shading correction may be omitted. The definition is also quite good.

When the two types of pick-up tube are compared, the quality of the picture produced by the vidicon is generally better. Optimum operating conditions are much more difficult to attain in the case of a three-tube image orthicon colour camera, and, in addition, this tube is more sensitive to temperature variations and microphony. Last not least there is the difference in size. Fig. 12 shows a vidicon and an image orthicon with its focussing and deflection coils. Clearly it will become easier to construct a colour camera with smaller dimensions when such a vidicon can be used.

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Transmission of colour television signals

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Summary

The paper discusses the transmission of colour television signals according to the NTSC system. The choice of the chrominance signals, their bandwidths and of the subcarrier frequency is discussed. The consequences of the method of gamma correction and of deviations from the constant-luminance principle are considered. The significance of the statistics of the chrominance signal is pointed out.

1. Introduction.

The companion paper by de Vrijer explained the principles of colour television. It was stated that full information about the luminance and the colour of each part of the televised scene can be provided by three independent data. As a consequence the output of a signal source for colour television delivers three independent signals, which are commonly termed the red, green and blue signals and are denoted by the symbols R, G and B.

It is the task of the transmission system to combine these three primary signals in a suitable manner into one composite signal which can be transmitted by a radio frequency transmitter.

Before entering into the details of this transmission problem it seems worth-while to survey the requirements to be met by the transmission system.

These are:

1. The colour-television receiver shall be able to present a good colour reproduction of the original scene.
2. For economical reasons the receiver has to be as simple as possible.
3. The transmission system has to be compatible with monochrome television, that is: a normal black-and-white receiver tuned to the colour broadcast shall reproduce it as a normal

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black-and-white transmission. On the other hand a colour television receiver has also to be usable for monochrome transmissions.

4. In view of this compatibility requirement and in view of considerations of bandwidth economy the signal has to be such that it can be transmitted within the existing transmission channels. The planning of these channels is based on normal monochrome transmission; for the 625-line system their bandwidth is 7 Mc/s, the spacing between the vision and sound carriers being 5.5 Mc/s.
   Hence about 5 Mc/s is available for the composite video signal.

At first glance these requirements must seem rather exacting and in some respects conflicting, but it will be shown that a good compromise is quite well possible.

2. Basic principles of colour-television transmission.

2.1. *Luminance and chrominance signals.*

Let us first consider the nature of the information which has to be transmitted. In colour television we have the luminance and the colour of each part of the scene, while in monochrome television only information about the luminance is transmitted. As we have seen, full information concerning colour and luminance of the scene can be represented by three independent video signals. It will be clear that no information gets lost if we transmit three independent combinations of these primary colour signals instead of the signals themselves. More specifically it is feasible to choose one of these combinations in such a manner that it represents the luminance of the scene. The colour has than to be defined by two other combinations of the primary colour signals. Working this way and transmitting the luminance signal in quite the same manner as in monochrome television we gain two important advantages. First we fulfil our requirement of compatibility: a normal monochrome receiver will use the luminance signal in the normal way, hence a normal black-and-white picture is reproduced. Second we can use to good advantage a remarkable property of the human eye. A picture in which only the luminance information is displayed sharply whereas the colour information is displayed with much less sharpness is appraised as sharp by the eye. This property can easily be shown by a simple experiment; in fact it has already many other applications, e.g. the well-known coloured picture postcards of landscapes and beach-scenes. These are commonly normal black-and-white pictures to which very roughly some colour is added. The result may be liable to discussion from the aesthetic point of view, but its main drawback is certainly not a lack of sharpness.

If the three primary colours of the system are chosen as explained in the paper by de Vrijer, the luminance of the scene is given by the signal:

$$Y = 0.59\,G + 0.30\,R + 0.11\,B \tag{1}$$

This signal is transmitted in the same way as a normal monochrome signal, i.e. it is transmitted with the full 5 Mc/s bandwidth. The two remaining signals determine only the colour to be reproduced and hence can be transmitted with a much smaller bandwidth according to the principle mentioned above.

2.2. *The dot-interlace principle.*

At this stage we must look for a method to find room for these two narrow-band signals in the video band which is seemingly already fully occupied by our luminance signal. For this purpose we can make use of the "dot-interlace principle". According to this principle the disturbing effect caused by a foreign signal in the video band is only small if the disturbing frequency is an odd multiple of half the line frequency, as is easily seen from Figure 1. In this figure representing part of a television scanning pattern with a disturbing frequency being present which is an odd multiple of half the line frequency, the letters A denote the dots produced by the disturbing signal in the first field of scanning. The letters B denote the dots occuring in the second field and in the same manner the letters C and D represent the dots occuring in the third and the fourth field. Hence a complete cycle of the disturbing pattern takes up two full frame scannings. As is easily seen from our figure the disturbing patterns are opposite each other in successive lines and in successive frames. Because of the integrating properties of the eye the light impressions of successive lines and frames

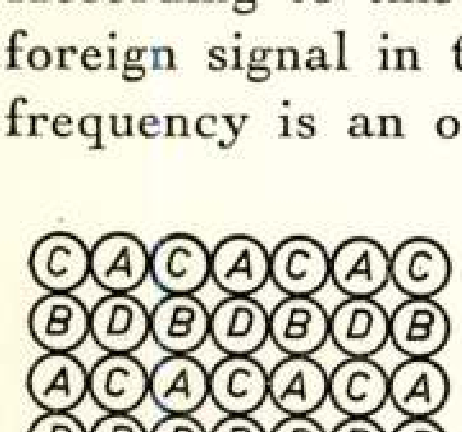

Fig. 1
Scanning pattern for odd multiple of half the line frequency.

due to the disturbing signal will more or less compensate each other. Hence a spurious signal of such a frequency is only slightly objectionable. This enables us to introduce one or more subcarriers into the luminance signal, provided their frequencies are odd multiples of half the line frequency. Modulation of our colour signals onto such subcarriers allows us to transmit these signals within the frequency band of the luminance signal.

3. The NTSC-transmission system.

3.1. *Modulation and demodulation of the subcarrier signal.*

The method of introducing one or more subcarriers into the luminance signal in the manner described above is employed in almost all known experimental transmission systems for colour television. These systems differ in the number of subcarriers employed, the way they are modulated and the choice of the modulating colour signals. Rather than giving a survey of all transmission systems investigated until now we shall confine ourselves in this paper to a more detailed discussion of the most developed system which is in our opinion the best one. This is the NTSC-system [7]) which was developed in the U.S.A. in a combined effort of all leading industries in the field, who for this purpose created the National Television System Committee. Of course this system is adapted to the American black-and-white standard but the underlying principles can be applied in European versions of this system as well.

This system employs only one subcarrier modulated by both colour signals. We shall denote these signals by I and Q and for the present not consider in what manner they are composed of the primary colour signals.

Let the angular frequency of the subcarrier be ω. The subcarrier is amplitude-modulated by the first signal I, the subcarrier being suppressed. This can be achieved by employing a balanced modulator; at the output of the modulator we obtain the signal $I \cos \omega t$. The second colour signal Q is modulated in quadrature onto the subcarrier in the same manner, hence the signal $Q \sin \omega t$ is obtained. Adding these signals together we get the composite chrominance signal:

$$I \cos \omega t + Q \sin \omega t = \sqrt{I^2 + Q^2} \sin (\omega t + \arctan \frac{I}{Q}) \qquad (2)$$

From this formula we see that we have modulated the subcarrier in phase as well as in amplitude by the colour signals. To demodulate the composite signal we multiply it in the receiver by $\sin \omega t$ and by $\cos \omega t$, respectively, and find:

$$(Q \sin \omega t + I \cos \omega t) \sin \omega t = \tfrac{1}{2} I \sin 2 \omega t + \tfrac{1}{2} Q - \tfrac{1}{2} Q \cos 2 \omega t \quad (3)$$

After filtering out the terms with double subcarrier frequency we get $\frac{1}{2} Q$.

In the same manner multiplication by $\cos \omega t$ yields:

$$(Q \sin \omega t + I \cos \omega t) \cos \omega t = \tfrac{1}{2} Q \sin 2 \omega t + \tfrac{1}{2} I + \tfrac{1}{2} I \cos 2 \omega t) \quad (4)$$

Hence, after filtering we obtain $\frac{1}{2} I$.

To perform the above operations, which are commonly termed synchronous detection, we have to produce in the receiver a subcarrier which is exactly synchronous with the subcarrier in the transmitter. For this purpose a synchronizing signal is introduced into the composite video signal, consisting of about 9 cycles of subcarrier frequency with known phase and amplitude. This synchronizing signal (usually named the "colour burst") provides the reference phase for the local oscillator in the receiver. It is applied in the line-blanking interval after the line-synchronizing impulse. Thus the composite video signal will assume the shape given in Fig. 2.

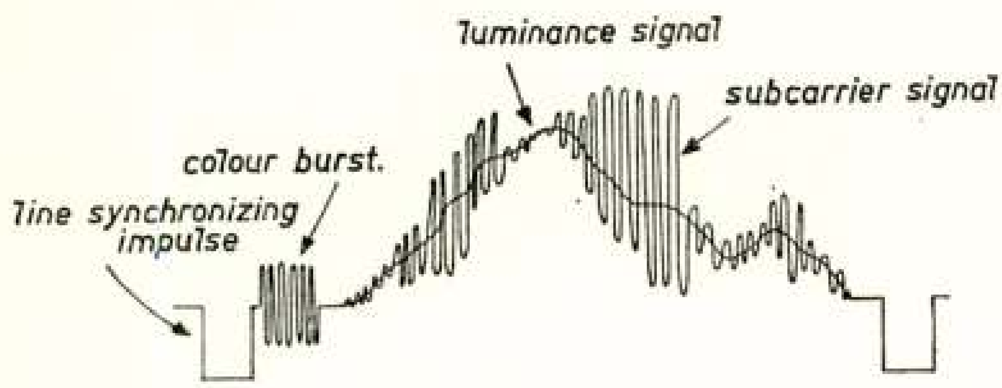

Fig. 2
Composite colour-television signal.

3.2. *Choice of the colour signals.*

3.2.1. General requirements. At this stage the problem of selecting the signals I and Q arises. First of all we have to bear in mind that in the receiver we have to retransform the signals Y, I and Q into the primary colour signals R, G and B. To get a simple transformation it is advantageous to choose for I and Q as well as for Y linear combinations of the primary colour signals. As we have seen Y is transmitted with full bandwidth whereas I and Q are narrow-band signals. Assuming equal bandwidths for the I and Q-signals to facilitate our considerations, we can split up each of the primary colour signals R, G and B into a low-frequency part, which is transmitted by the signals I and Q as well as by the luminance signal Y, and a high-frequency part which is transmitted by the luminance signal only. Denoting the low-frequency parts by the subscript L and the high-frequency parts by the subscript H, we can write:

$$Y = Y_L + Y_H = 0{,}30\,(R_L + R_H) + 0{,}59\,(G_L + G_H) + 0{,}11\,(B_L + B_H) \tag{5a}$$

$$I = \alpha_1 R_L + \beta_1 G_L + \gamma_1 B_L \tag{5b}$$

$$Q = \alpha_2 R_L + \beta_2 G_L + \gamma_2 B_L, \tag{5c}$$

where the symbols α and β denote numerical constants. In the receiver we obtain the low-frequency parts of R, G and B by weighted addition of Y_L, I and Q. To the three primary colour signals obtained in this way we add the high-frequency part of the luminance signal Y_H, which is commonly termed the "mixed highs". Hence, to produce the primary signal R in the receiver we have to add to the luminance signal $Y_L + Y_H$ the signal: $R_L - Y_L$; to get B we have to add: $B_L - Y_L$ etc. However, it is desirable to avoid the necessity of having to form the separate parts Y_L and Y_H in the receiver, since this leads to rather severe complications. Formation of the signal Y_L from the luminance signal requires a filter whose amplitude response is exactly matched to that of the corresponding filters in the transmitter. This complication is avoided if the I and Q-signals are chosen in such a manner that it is possible to obtain in the receiver the signals $R_L - Y_L$, $B_L - Y_L$ and $G_L - Y_L$ without making use of the luminance signal Y, but using only the signals I and Q. Obviously to make this possible it is sufficient to choose for I and Q linear combinations of $R_L - Y_L$ and $B_L - Y_L$. If $R_L - Y_L$ and $B_L - Y_L$ are available we can obtain $G_L - Y_L$ from them quite simply, as this signal is already in itself a linear combination of $R_L - Y_L$ and $B_L - Y_L$.

Such a choice for the signals I and Q has an important second advantage. As was explained in the paper by de Vrijer the signal sources are arranged in such a manner that in reference white (Illuminant C) the three primary colour signals are equal in magnitude, hence for white we have $R = G = B = Y$. This means that for colourless parts of the scene $R - Y = B - Y = 0$ and hence also $I = Q = 0$. From expression (2) for the composite chrominance signal we see that the subcarrier amplitude then also equals zero. This means that we have no subcarrier signal in colourless parts of the picture, while the subcarrier amplitude

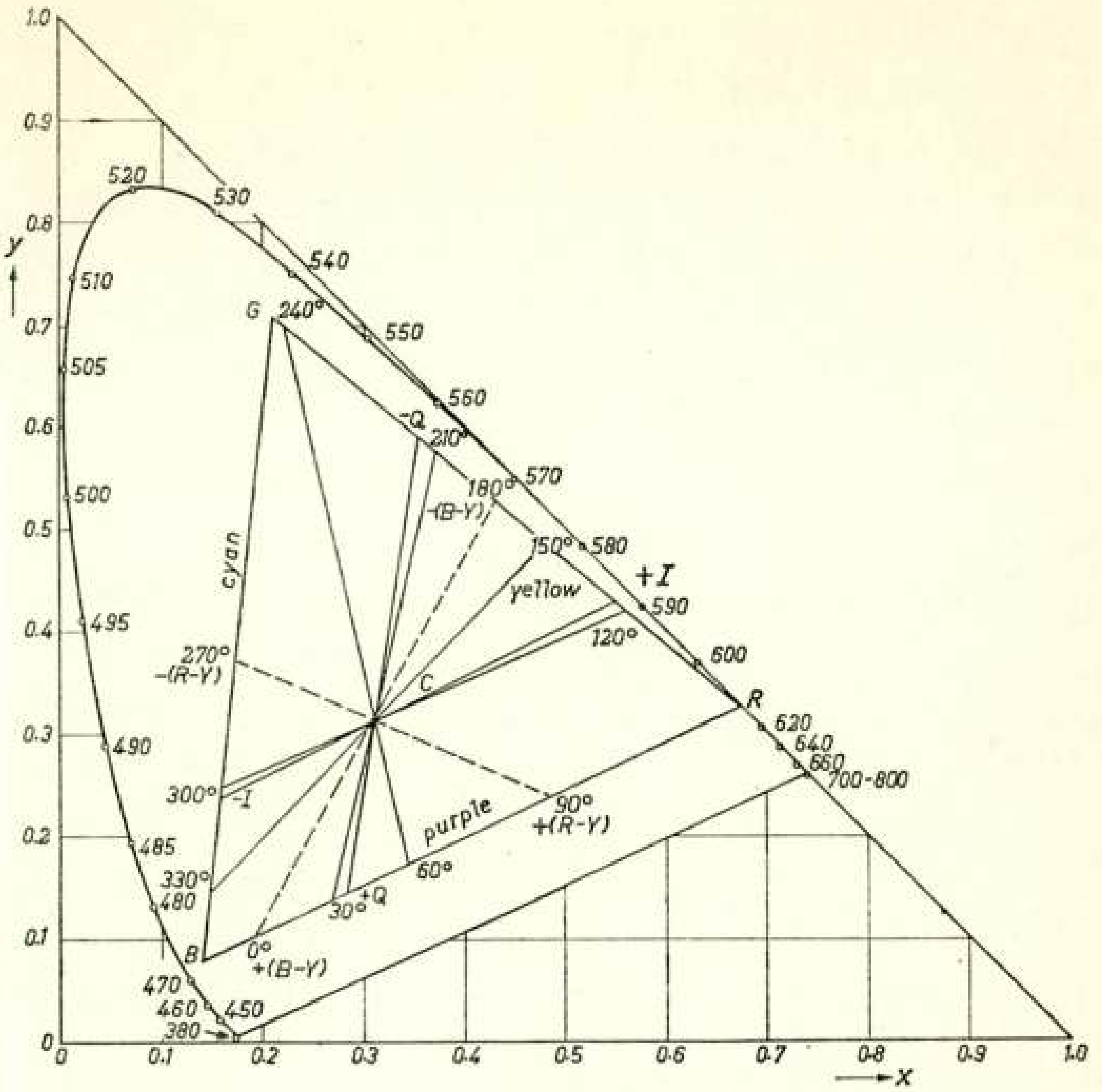

Fig. 3

Chromaticity diagram showing colour-subcarrier phase as a function of reproduced chromaticity ($\gamma = 1.0$).

increases with increasing saturation of the colour to be transmitted. The phase of the subcarrier signal depends on the hue of the colour. Assuming the transmission system to be linear and determining the contours of equal subcarrier phase in the chromaticity diagram we obtain Fig. 3.

The assumption of linearity of the system is an oversimplification as the actual system is on purpose made non-linear to correct for the non-linearity of the display tube; if we allow for this non-linearity in the calculation the final result differs considerably from the simple presentation of Fig. 3. In a subsequent part of this paper we shall discuss this problem in more detail.

Returning to our discussion on the choice of the colour signals we shall have to deal with one problem remaining at this stage, i.e. the most suitable choice of linear combinations of R-Y and B-Y for the signals I and Q. When the signals I and Q are modulated with equal bandwidths onto the subcarrier, their composition is based on considerations concerning:

1. The allowable overswing of the subcarrier signal, that is: the amount by which the subcarrier signal surpasses the peak white and blanking levels of the signal.
2. The dependence of the hue of the colour to be reproduced on the phase of the subcarrier signal. The dependence has to be such that in all parts of the chromaticity diagram the proper relation exists between subcarrier phase and hue, viz. that the hue-differences which are just noticeable to the eye correspond to the same phase deviations.

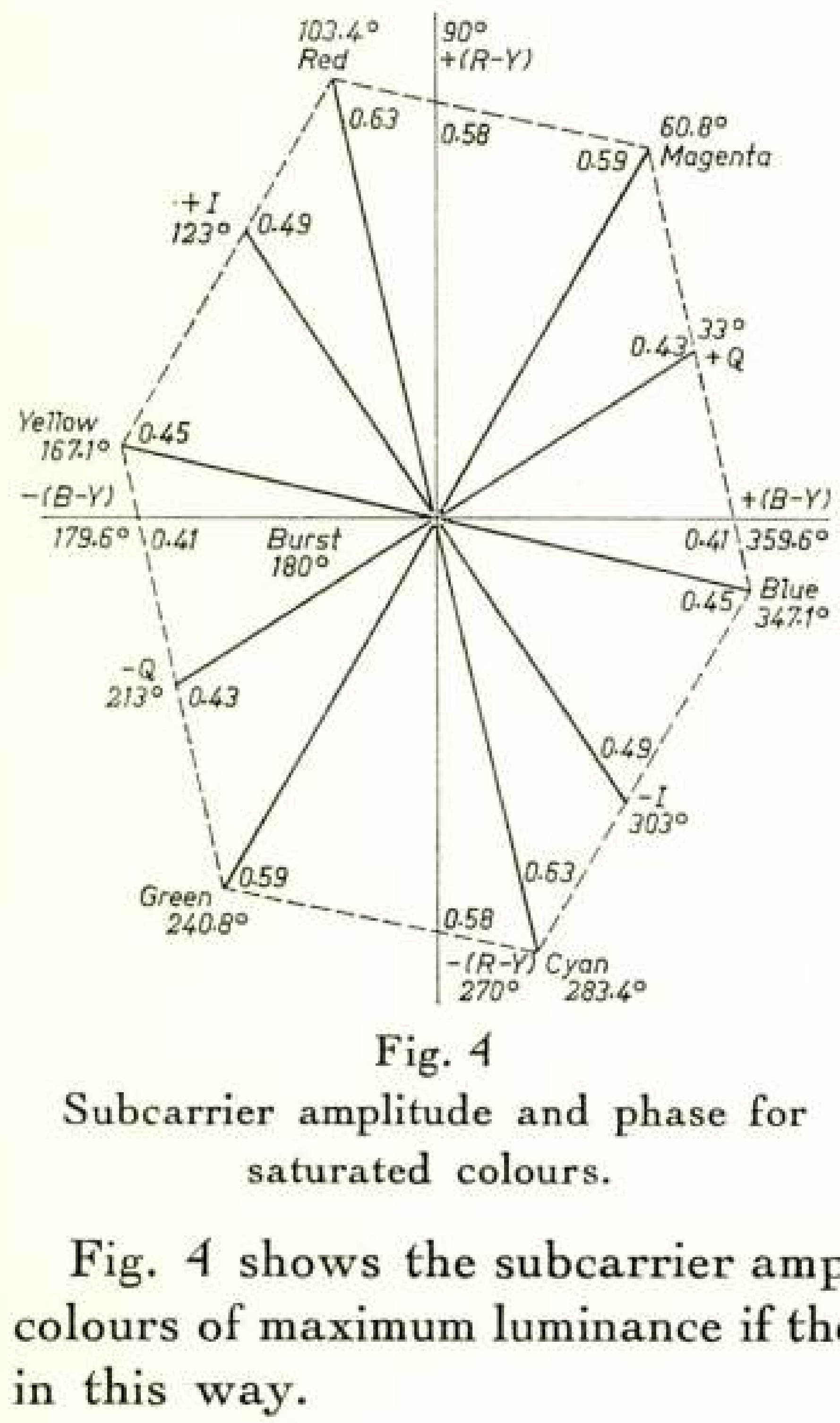

Fig. 4
Subcarrier amplitude and phase for saturated colours.

For a detailed discussion of this problem we refer to the literature on the subject [10]). Investigations based on the considerations mentioned above lead to the following choice of both colour signals to be modulated in quadrature onto the subcarrier:

$$0.49\,(B - Y) \text{ and}$$
$$088\,(R - Y).$$

Hence, the composite colour-television signal can be written:

$$S = Y + 0.49\,(B - Y) \sin \omega t + 0.88\,(R - Y) \cos \omega t \quad (6)$$

Fig. 4 shows the subcarrier amplitude and phase for saturated colours of maximum luminance if the colour signals are composed in this way.

3.2.2. Unequal bandwidths of I and Q-signals. In the case of different bandwidths of the I and Q-signals the problem is more difficult. The same considerations as in the case of equal bandwidths apply, but in addition the question arises as to how the extra bandwidth has to be employed. Before dealing with this problem we shall first show that it is possible to transmit the signals I and Q with different bandwidths. This is easily seen if we observe more closely the process of synchronous detection, which is expressed in the relations (3) and (4). These relations indicate that for synchronous detection of the colour signals both sidebands of the modulated signals must be present. If one sideband of, e.g., the I-signal is suppressed we find full crosstalk of the I-signal into the Q-channel. Let us now consider the case of different bandwidths for the I and Q-signals. Let f_I be the cut-off frequency of the I-channel and f_Q that of the Q-channel. Furthermore we suppose $f_I > f_Q$. We now apply double-sideband modulation to the Q-signal but vestigial-sideband modulation to the I-signal in such a manner that all components of this latter signal which contain frequencies up to f_Q are double-sideband modulated, while the remaining components, containing frequencies between f_Q and f_I, are single-sideband modulated. In that case there will not be any crosstalk of the Q-signal into the I-channel, whereas only those components of the I-signal which contain frequencies between f_Q and f_I will crosstalk into the Q-channel. However, the Q-signal itself does not contain these frequencies because f_Q is the upper limit of its bandwidth, hence the crosstalking components from the I-signal can easily be removed from the Q-signal by a simple low-pass filter with cut-off frequency f_Q.

If the composite colour-television signal is composed along the lines set forth until now, its video-spectrum will be as presented in Fig. 5, where it is compared with that of normal monochrome signal, the only difference being the presence of the subcarrier signal.

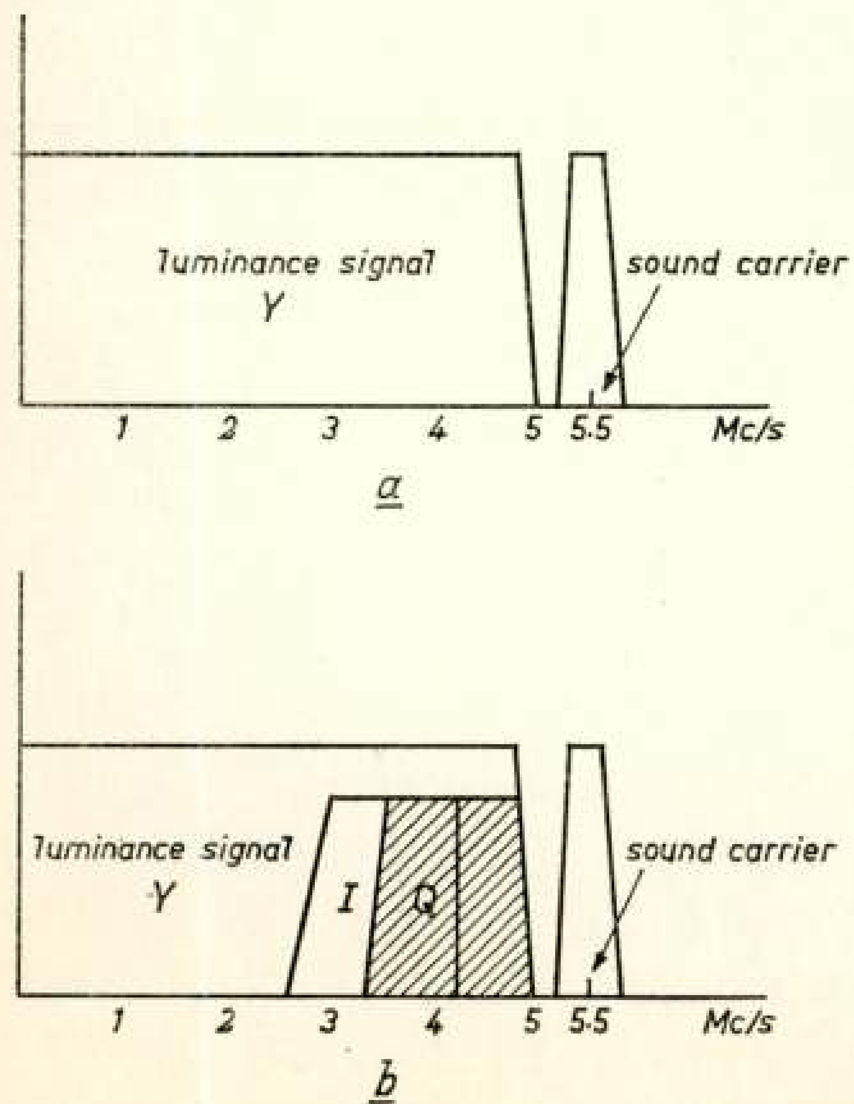

Fig. 5

a) Monochrome-television channel.
b) Colour-television channel.

Having thus proved the possibility of employing different bandwidths for the I and Q-signals we shall continue our discussion of the choice of these signals and their bandwidths.

It will be clear that in spite of the employment of dot-interlace a certain amount of crosstalk between the luminance signal and the subcarrier signal will be observable, due to the non-ideal integrating properties of the eye and the non-linear behaviour of the transmission system, which causes the subcarrier signal to be rectified. This remaining crosstalk is stronger as the subcarrier frequency is located lower in the video band and the bandwidths of the colour signals are larger. On the other hand it will be clear that a too severe bandwidth limitation of the subcarrier has to be avoided, too, as this will cause a lack of sharpness of the picture. Hence a good compromise has to be found between these conflicting requirements. This compromise depends among other things on the available video bandwidth, so it has to be determined anew for each individual adaptation of the NTSC-system to an existing black-and-white standard. The only method to find optimum compromise is to carry out suitable experiments [2] [3] [7] [11]. Such experiments have to include an investigation into the effects of bandwidth limitation in the I and Q-channels on the quality of the reproduced picture, and into the mutual crosstalk between luminance and colour information.

The final result depends on, among other things, the non-linear behaviour of the transmission system. We shall therefore discuss the results of such experiments after the discussion of the non-linear behaviour of the system. For the present we shall give the final result of the experiments supposing the total available bandwidth to be 5 Mc/s, as is the case in the European 625-line system. A good compromise is obtained if the bandwidth of the I-signal is about 1.3 Mc/s and that of the Q-signal about 0.5 Mc/s, if we compose the I and Q-signals in the same manner as in the American system, that is:

$$I = -0.28\,G + 0.60\,R - 0.32\,B \tag{7a}$$

$$Q = -0.52\,G + 0.21\,R + 0.31\,B \tag{7b}$$

3.3. *Block diagram of the complete transmission system.*

We are now able to give a block diagram of the complete transmission system. The device which forms the composite NTSC signal from the three primary colour signals is commonly termed the encoder. Its block diagram is given in Fig. 6a. The input signals R, G and B are linearly transformed into the signals Y, I and Q by the matrix circuit. After having passed different low-pass filters the I and Q-signals are modulated in quadrature onto the subcarrier.

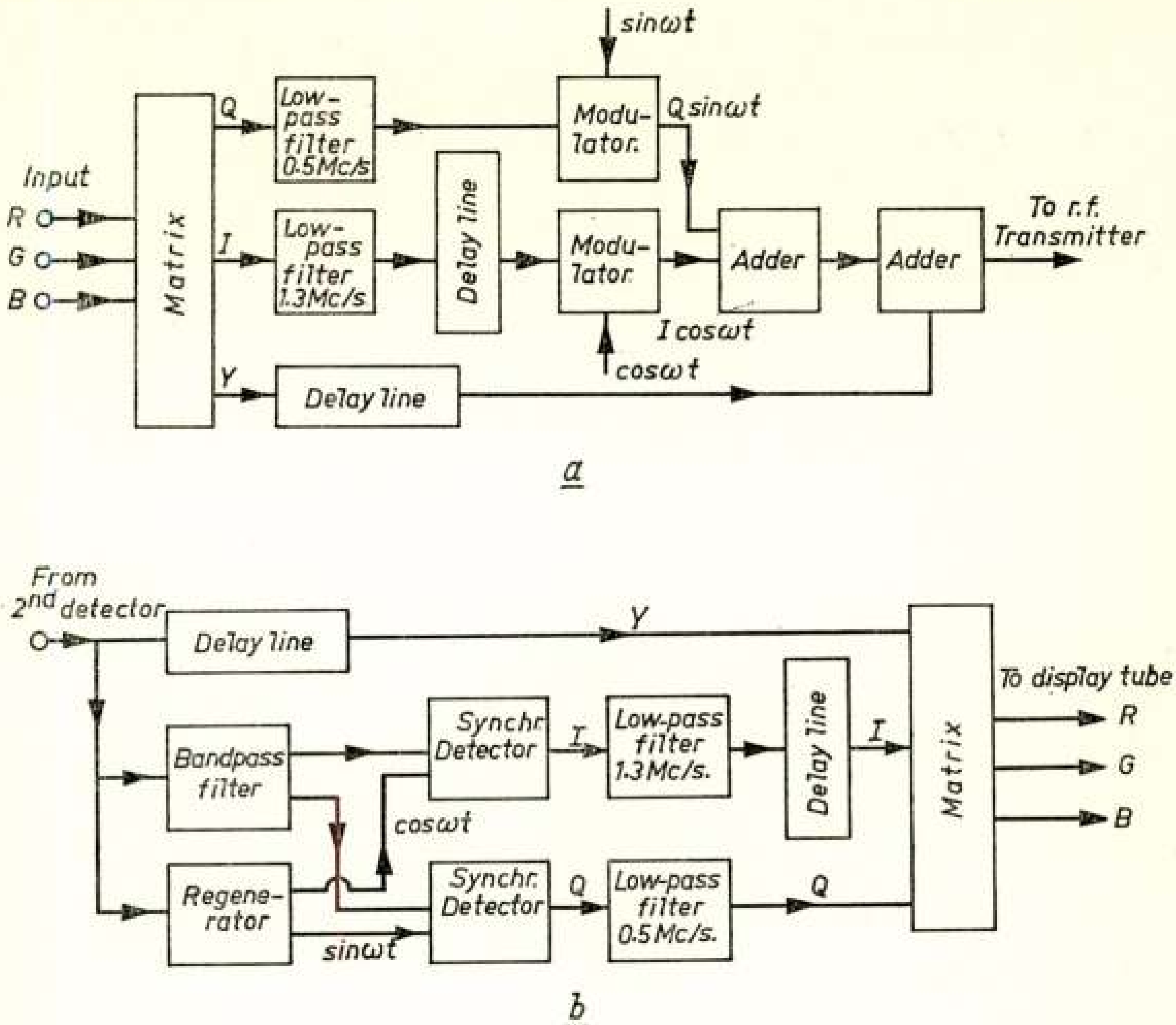

Fig. 6

a) Block diagram of encoder.

b) Block diagram of decoder.

The modulated signals are added together, thus forming the composite subcarrier signal, which is finally added to the luminance signal. In the luminance channel and in the *I*-channel delay lines are provided to match the different time delays of the various filters.

The composite signal is transmitted by a r.f. transmitter, received, and detected by the second detector in the receiver. The signal is then decoded, that is: from the composite signal the three primary colour signals are re-formed. The decoder section of the receiver is given in Fig. 6b. The subcarrier signal is taken from the composite signal with the aid of a band-pass filter. The burst signal is keyed out and employed in the regenerator which provides the demodulating subcarrier, which is fed to two synchronous detectors, producing the *I* and *Q*-signals from the composite subcarrier signal. After filtering these signals are fed to a matrix circuit together with the luminance signal *Y*. The output signals *R*, *G* and *B* of this circuit are fed to the display tube.

4. Gamma correction and non-linear behaviour of the system.

4.1. *The constant-luminance principle.*

For convenience we have so far ignored all non-linearities in the system. It will now be necessary to include this extra complication of the transmission problem into our considerations.

For a linear system the „constant-luminance principle" is valid. A colour-television system is said to be designed in accordance with this principle if none of the signals in the narrow-band chrominance channel contributes to the luminance of the reproduced picture. Among other things this means that the bandwidth limitations in the chrominance channel do not affect the luminance rendering [4]) [6]) [9]).

4.2. *Gamma correction in colour-television.*

As is well-known all display devices show non-linear characteristics. The relation between beam current and driving voltage is given by an expression of the type: $i = kV^{\gamma}$, where k denotes a numerical constant and γ is the "gamma" of the tube, which is usually about 2.2. If we want to avoid non-linear circuits in the receiver which correct this non-linearity, we have to introduce the correction at the camera end. In monochrome television this is accomplished in a simple manner by transmitting the signal $Y^{1/\gamma}$ instead of the signal Y, but in colour-television the matter is considerably more complicated. If we want to adhere to the constant-luminance principle, we must employ a luminance signal which carries all luminance information. The luminance of the original scene can be represented by:

$$L = 0.59\, G + 0.30\, R + 0.11\, B \tag{8}$$

Hence, to satisfy the constant-luminance principle the luminance signal has to be of the general type:

$$Y'' = p\,(0.59\, G + 0.30\, R + 0.11\, B)^{q}, \tag{9}$$

where p and q are constants. One might choose $q = 1/\gamma$, γ again being the gamma of the picture tube. In order to produce the correct picture this tube has to be fed with signals of the type: $G^{1/\gamma}$, $R^{1/\gamma}$ and $B^{1/\gamma}$. It will be clear that, supposing the luminance signal to be of the form (9), it is impossible to find I and Q-signals which, together with the signal Y'', can be used to form the signals $G^{1/\gamma}$, $R^{1/\gamma}$ and $B^{1/\gamma}$, without requiring non-linear elements in the receiver. The application of non-linear elements has to be avoided, if possible, as it complicates the receiver severely. For this reason a compromise is accepted in practice: The gamma correction is applied to the primary colour signals, rather than to the luminance signal. This means that the signal sources are made to deliver directly the signals $G' = G^{1/\gamma}$, $R' = R^{1/\gamma}$ and $B' = B^{1/\gamma}$. These signals are treated in quite the same manner as the signals R, G and B in the linear system as described above. Hence at the output terminal of the matrix network in the receiver we obtain the signals $G^{1/\gamma}$, $R^{1/\gamma}$ and $B^{1/\gamma}$, which are precisely the signals needed to drive the picture tube. However, this implies that the luminance signal can be written:

$$Y' = 0.59\, G^{1/\gamma} + 0.30\, R^{1/\gamma} + 0.11\, B^{1/\gamma} \tag{10}$$

By comparison with (9) we see immediately that this signal does not satisfy the constant-luminance principle. Nevertheless the proper signals are fed to the picture tube, hence the proper luminance values are reproduced. This means that part of the luminance information has reached the display device via the

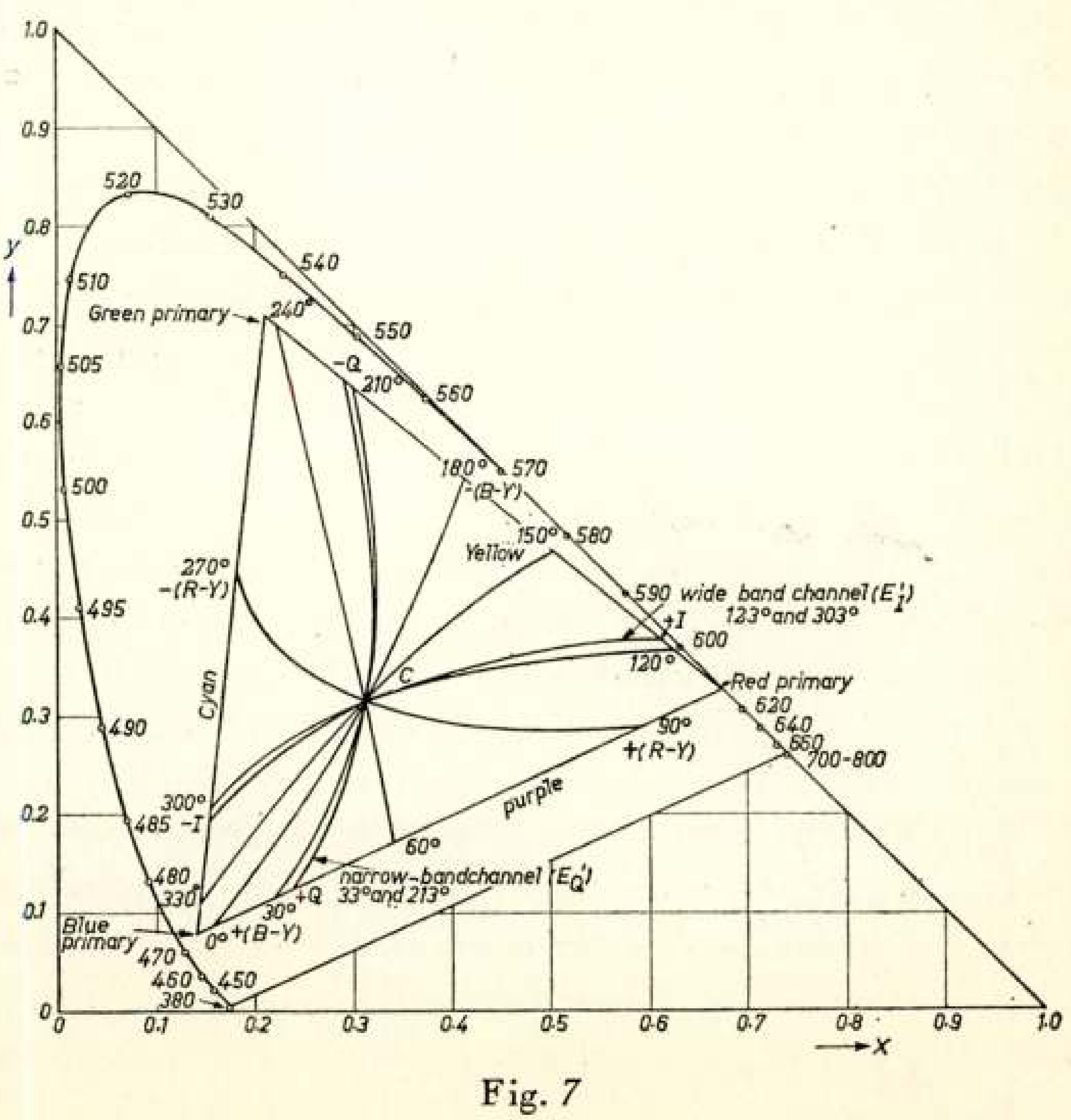

Fig. 7

Colour-subcarrier phase as a function of reproduced chromaticity ($\gamma = 2.2$).

narrow-band chrominance channel. Assuming $\gamma = 2$ to simplify the calculations, one finds that the reproduced luminance is given by:

$$L = Y'^2 + 0.46 I'^2 + 0.15 I' Q' + 0.67\, Q'^2, \tag{11}$$

where Q' and I' denote the I and Q-signals according to (7) but composed of the gamma-corrected primary colour signals R', G' and B'.

As a consequence, the contours of equal subcarrier phase in the colour triangle will differ from those for the linear transmission system. They are presented in Fig. 7 for $\gamma = 2.2$.

4.3. *Statistical properties of the chrominance signal.*

From (11) we see that the subcarrier signal is elliptical, that is: the Q-signal contributes more to the luminance than the I-signal. Hence, we might expect from (11) that the bandwidth limitations in the Q-channel will have a greater bearing on the rendering of the luminance detail than those of the I-channel. Remarkable enough experiments show that in practice this holds only for artificial signals. If normal picture material is used the result is even opposite to what is to be expected from (11). A closer investigation into this phenomenon shows that it is caused by the statistical distribution of the colours in normal scenes. If we demodulate the subcarrier signal, according to Fig. 4, with different phases of the demodulating subcarrier thus obtaining different linear combinations of $0.49\,(B - Y)$ and $0.88\,(R - Y)$, then we find that the average signal content is maximum if the phase of the demodulating subcarrier is at an angle of about 30° with the $(R - Y)$ phase, whereas a minimum occurs when this angle is about 100° larger. The average signal contents for the maximum and minimum axes are in the proportion of about 3 : 1. This result provides a sound reason for encoding the colour information in such a manner that the composing signals I' and Q' have different bandwidths. In that case the I-signal, which has the larger bandwidth, has to correspond to the colour information of the axis of maximum signal content as found above. The other signal has to be in quadrature with this signal and hence carries the colour information corresponding to an axis, close to the minimum signal-content axis.

4.4. *The composite colour-television signal.*

For reasons mentioned before the signal has to be such that the vector presentation of Fig. 4 applies, that is: $R-Y$ and $B-Y$ must be modulated in quadrature onto the subcarrier. However, this does not imply that the signals $R-Y$ and $B-Y$ themselves have to be used in the modulation process. As is readily seen from Fig. 4 an infinite number of sets of axes which are in quadrature to each other can be given. According to our experimental results the most suitable set of these axes is at an angle of about 30° with the set $R'-Y'/B'-Y'$. In the American system this angle is 33°. As was explained before the I-signal is vestigial-sideband modulated on the subcarrier while the Q-signal is double-sideband modulated. The composite signal can therefore be written:

$$S = Y' + Q' \sin(\omega t + 33°) + I \cos(\omega t + 33°)$$

At frequencies below f_Q we have double-sideband modulation for I' as well as for Q', hence at these frequencies the given exprsssion has to be equivalent with our original expression (6), or:

$$S = Y' + 0.49\,(B' - Y') \sin \omega t + 0.88\,(R' - Y') \cos \omega t. \quad (13)$$

To be in accord with these equations the I' and Q'-signals have to be:

$$\begin{aligned} Q' &= 0.41\,(B' - Y') + 0.48\,(R' - Y') = \\ &\quad - 0.28\,G' + 0.60\,R' - 0.32\,B' \end{aligned} \quad (14a)$$

$$\begin{aligned} I' &= -0.27\,(B' - Y') + 0.74\,(R' - Y') = \\ &\quad - 0.52\,G' + 0.21\,R' + 0.31\,B' \end{aligned} \quad (14b)$$

These equations are equivalent with equations (7), given earlier.

4.5. *Bandwidth of colour signals and crosstalk phenomena.*

The choice of different bandwidths for the I' and Q'-channels being motivated, we still have to find the optimum values for these bandwidths. This has to be done by carrying out suitable experiments. We shall not describe such experiments in detail [1] [2] [11] but confine ourselves to the most important results.

If we transmit the luminance signal and the subcarrier signal separately, thus eliminating all chances that crosstalk occurs between these signals, we can obtain data about the visible effects of bandwidth limitation in the colour channels. As already mentioned these effects include luminance errors as well as errors in the colour reproduction, due to the lack of constant luminance in the system. On the other hand experiments can be carried out which permit the investigation of the visible effects of mutual crosstalk between the luminance signal and the modulated subcarrier signal. As to the crosstalk of the subcarrier signal into the luminance channel: due to non-ideal integrating properties of the eye and the rectification of the subcarrier signal caused by the non-linear characteristics of the picture tube, we cannot avoid having to use a subcarrier suppression filter in the luminance channel of the receiver. It turns out, however, that it is not necessary to suppress the subcarrier signal completely. If a relatively narrow band around the subcarrier frequency is removed by filtering, the remaining crosstalk caused by the sidebands of the subcarrier signal is only slightly visible, and less so as the subcarrier frequency is located higher in the video band.

As to the crosstalk of the luminance signal into the subcarrier channel: the high-frequency components of the luminance signal in the neighbourhood of the subcarrier frequency are treated by the subcarrier demodulators as sidebands of the subcarrier signal and hence give rise to spurious output signals of these demodulators. The magnitudes of these spurious signals may be assumed to increase roughly proportional with the bandwidth of the I and Q-channels. The experiments show that these spurious signals are mainly visible by their luminance contribution. We can see from (11) that the luminance contribution of signals in the Q-channel is considerably larger than that of signals in the I-channel. As the luminance contribution of the signals is approximately proportional to the square of their amplitude it is roughly proportional to the square of their bandwidth. On the other hand we have seen that the information content differs considerably for both colour signals. Combining the experimental results on the statistics of colour signals and on the phenomena of mutual crosstalk between luminance and chrominance signals we may draw the following important conclusion: *For either colour signal there exists an optimum bandwidth. If one increases the bandwidth beyond this value the useful information gained by increasing the bandwidth is of minor practical importance whereas the crosstalk of the luminance signal into the chrominance channel increases rapidly.*

Experimentally it was found that this optimum bandwidth is about 0.5 Mc/s for the Q-signal and 1.3 Mc/s for the I-signal. For the 625-line television standard a subcarrier frequency in the neighbourhood of 4.5 Mc/s is, therefore, to be preferred. The most suitable frequency in this region is 4.429687 Mc/s, which equals $567 \times \frac{1}{2} f_L$, f_L being the line frequency. Since $567 = (3^4 \times 7)$ does not contain larger factors we can use simple frequency dividers.

4.6. *Visibility of the luminance errors.*

Finally we shall discuss briefly the consequences of the lack of constant luminance in the system. As we saw before part of the luminance information is transmitted by the narrow-band chrominance channel. This causes distortions in the luminance rendering, viz. at sharp transients in saturated colours, generally known as "luminance notches"[9]). These luminance notches are very pronounced in artificial pictures, such as colour-bar signals. However, in normal pictures these effects are only seldom visible. This is again due to the statistical properties of the subcarrier signal.

From (11) one can easily see that the effects of constant-luminance failure occur mainly in saturated colours, that is: for large values of the subcarrier amplitude. Measurements carried out on the distribution of the amplitude levels of the subcarrier signal for normal pictures demonstrated that only during 1 % of the total time the subcarrier level surpasses half the maximum value. As the luminance errors depend roughly quadratically on the subcarrier amplitude, we can state that only during 1 % of the time the errors exceed 25 % of their maximum possible value.

From all these experimental results it will be clear that in the transmission of colour-television signals the statistical properties of the signals play an important role.

5. **Tritanopia of the eye to small objects versus signal statistics.**

It should be noted that the discussion of the transmission system as presented in this paper differs in some respects from that given in most publications on the subject. The choice of the colour signals and of their bandwidths is usually described as related to the tritanopia (a kind of partial colour-blindness) of the human eye to small objects [3] [8] [9]. However, the results of many experiments on the adaptation of this transmission system to the 625-line television standard have shown us that certain phenomena, which cannot be explained with the usual theory, fit in very well with the "statistical" approach. Of course we do not doubt the well-established fact of tritanopia of the eye to small objects; we merely believe that it does not provide an adequate explanation of the phenomena observed if bandwidth limitation is applied to the chrominance signals.

In many respects both theories lead to the same results, but in detailed discussions on the possibilities of improvements of the system both views lead to somewhat divergent conclusions.

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Colour television receivers

by H. Breimer *)

1. Introduction.

Since about 1951 many experimental receivers for a one-subcarrier colour television system have been constructed in the U.S.A., England, the Netherlands and other countries [1]) [2]) [3]). Many papers dealing with typical circuits for colour television receivers have appeared. However no combination of circuits has yet emerged that forms a completely satisfactory colour television receiver.

It is impossible in an article of practical size to compare the merits of the many circuits available for each of the colour television receiver functions.

In view of this situation the author feels free to describe the functions that should be incorporated in a colour TV receiver and to illustrate the descriptions with the circuits of the receiver designed at the Philips Research Laboratories at Eindhoven. The reader should bear in mind however that the detailed circuits are not representative of current design practice in the U.S.A.

2. Signal processing.

The signal decoding functions performed by the colour television receiver will be described with reference to the diagram in fig. 1. The blocks marked with an asterisk are not present in a monochrome receiver. The other circuits, though in principle equivalent to their counterparts in the monochrome receiver, have to handle a different type of signal.

Consequently, some of their properties are different. The NTSC type of colour television signal as produced by an encoder has been described in a companion paper by Davidse [4]). In fig. 5b of that paper is shown how the three signals Y, I

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and Q are placed in the available video band. It should be noted that there are two regions of large average signal energy surrounding respectively the picture carrier and the colour sub-carrier.

In a monochrome signal, large energy signal components are found only near the picture carrier for average programme material. The low frequency colour components, that are converted to the upper part of the video spectrum, form the second region with large radiated signal energy.

Consequently, errors of amplitude- and phase response in the high video frequency region of the i.f. amplifier are more

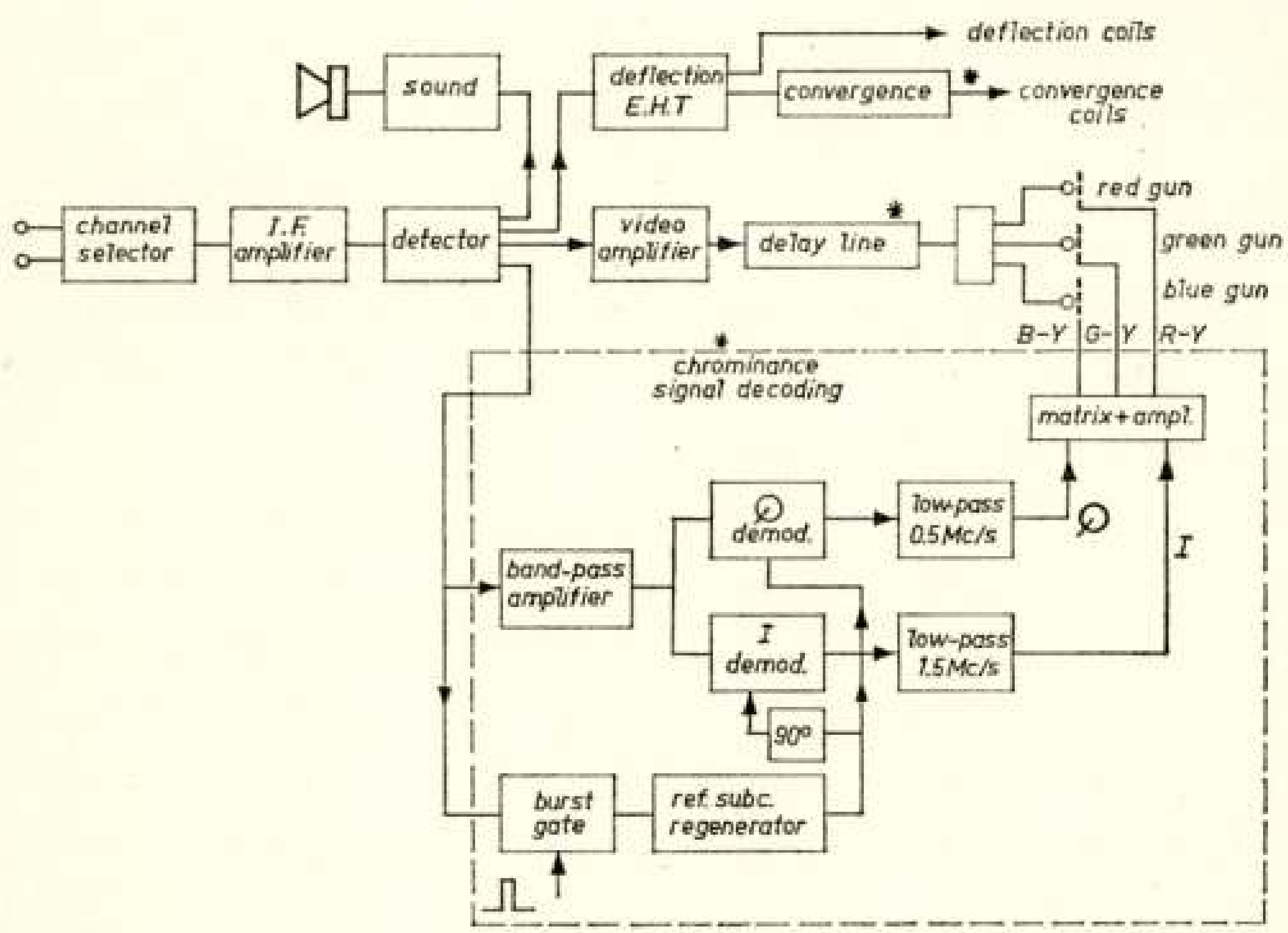

Fig. 1
Block diagram colour TV receiver.

serious than in monochrome receivers. They cause some colour transient defects, among which coloured smears are the most annoying.

A further potential source of poor colour performance is the detection process. When chrominance components with a relatively high amplitude are present at the detector, the latter hesitates which carrier to take as a reference for detection. The result is intermodulation of the luminance and chrominance signals [5]). The distorted luminance signal is characterized by a diminution of the signal level dependent on the amplitude of the chrominance signal. The chrominance signal distortion takes the form of an attenuation dependent on the luminance signal level. For a severe case, the distorted signals are compared to the original signals in fig. 2. The signal used for this illustration is one line of a test pattern consisting of coloured vertical bars of maximum luminance and saturation, flanked by a white and a black bar. The luminance distortion is particularly troublesome in a colour picture.

Fortunately many measures can be taken to restrict these distortions to a tolerable limit even under adverse conditions. One of them is to use two detectors designed in such a way that one detector circuit delivers only the luminance signal, and the other an undistorted chrominance signal. This is effected by partly suppressing the chrominance signal prior to detection of the luminance signal. In the channel prior to the

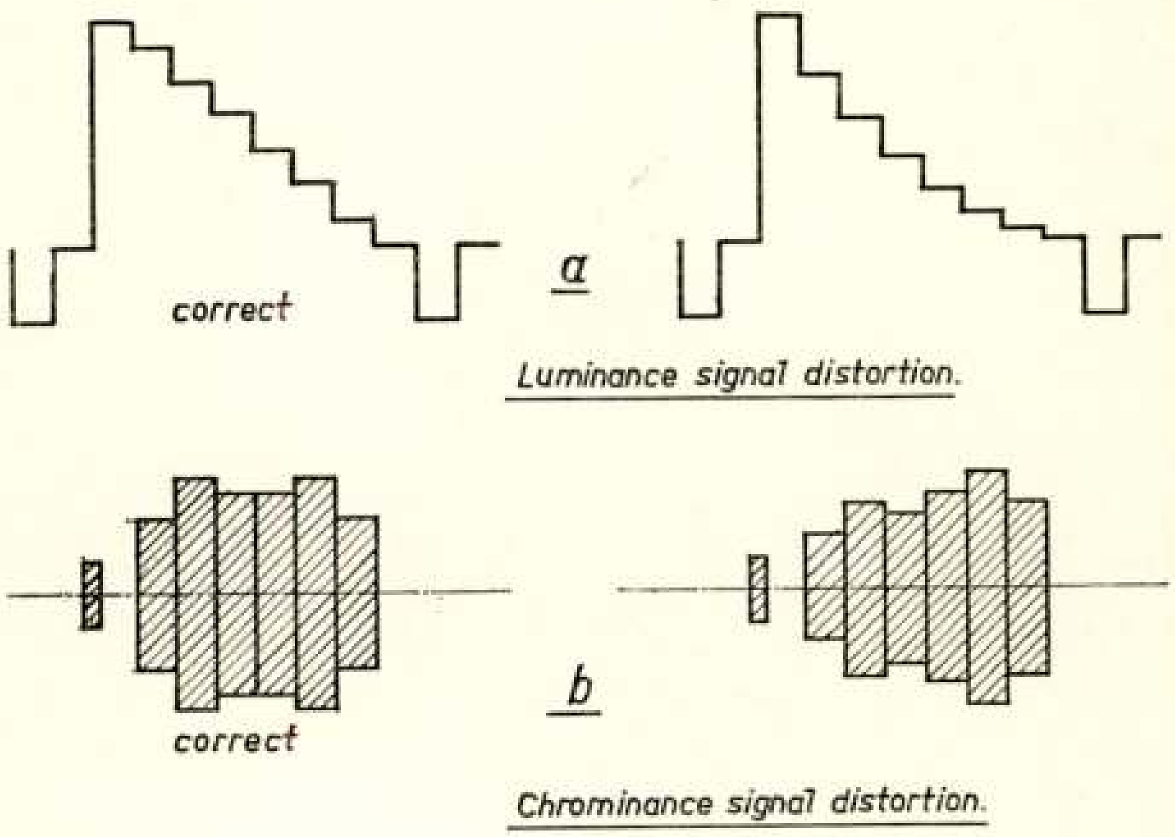

Fig. 2

other detector the picture carrier is enhanced relative to the chrominance signal. This measure, it is true, produces a distorted luminance signal together with the correct chrominance signal in this channel; in the subsequent chrominance band-pass amplifier, however, this luminance signal is rejected.

After detection the luminance signal is amplified and passed on to the kinescope guns via a distributing network. This network is necessary in order to correct for the unequal phosphor efficiencies. Without it no colourless gray scale could be reproduced.

At a suitable place in the luminance signal channel the insertion of a delay line is necessary in order to ensure exact time coincidence of the luminance- and colour-difference signals at the kinescope guns. The signal delays in the luminance and colour channels are different because of the different bandwidths in those channels. The amount of delay that should be introduced in the luminance channel depends on the receiver design and lies between 0.5 and 1 μ sec in most cases.

The display device used in almost every colour TV receiver today is the three gun shadow-mask kinescope. A great deal has been published about this tube and its construction [6] [7], so no further description will be given here.

In most receivers negative luminance signals $(-Y)$ are fed to the cathodes of the three guns, whilst colour difference signals $(R-Y, G-Y$ and $B-Y)$ are applied to the control electrodes. This results in the required modulation of the gun currents by the colour signals (R, G and B).

The maximum peak-to-peak voltage excursions of the Y, $R-Y$, $G-Y$ and $B-Y$ signals for one specific colour TV receiver design are tabulated below.

TABLE I

red gun	Y	65 V	(black-to-white)
	$R-Y$	91 V	
green gun	Y	47 V	
	$G-Y$	39 V	
blue gun	Y	40 V	
	$B-Y$	71 V	

In the chrominance signal channel the colour difference signals are derived from the original composite signal. The correct way to do this is first to obtain I and Q signals from two synchronous demodulators working along the appropriate axes.

If these demodulators are followed by low-pass filters with bandwidths equal to those used in the encoder, the original signals can be recovered without cross talk between the I and Q information. As the colour difference signals are linear combinations of the I and Q signals, they can be produced in a 'matrix' circuit performing very simple functions. This type of signal processing is used in our experimental receiver. In order to get a somewhat simpler and cheaper receiver the colour difference signals can be obtained directly from demodulators. In that case the demodulators should work along other axes, e.g. the $R-Y$ and $B-Y$ axes. The $G-Y$ signal, being a linear combination of $R-Y$ and $B-Y$, is then made in a simple matrix circuit.

Of course cross talk components will now be found in the region of 0.5 up to 1.5 Mc/s, owing to the single sideband modulation of I components in this region [5] [8]. However, the colour transient distortion produced by this form of cross-talk is so small that it is hardly noticeable in ordinary programme material.

3. Synchronous demodulators.

Three basic types of synchronous demodulator circuits applied in colour television receivers are drawn in fig. 3.

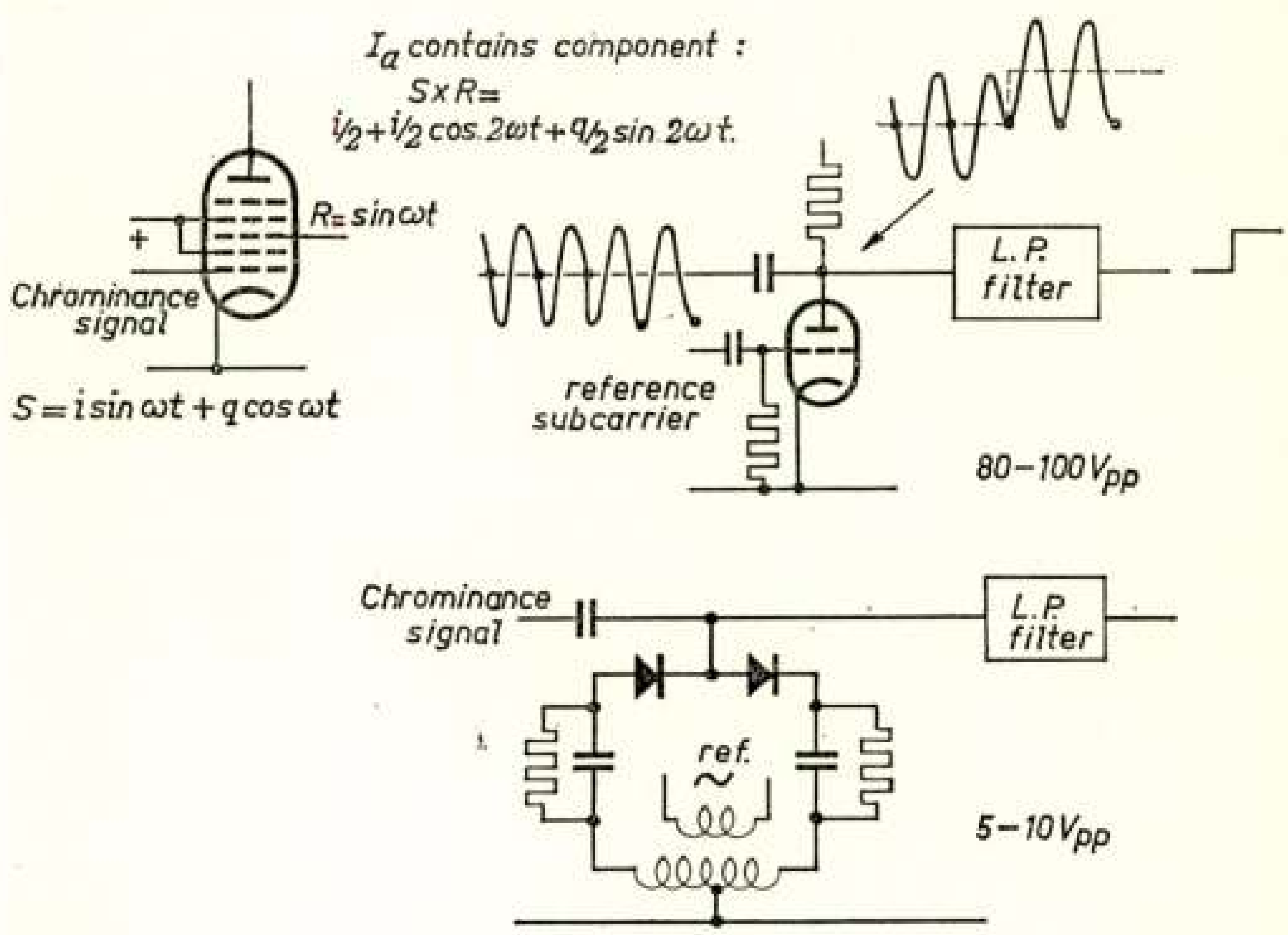

Fig. 3
Synchronous demodulators.

The pentagrid mixer of fig. 3a was used in the first receivers. The useful video output is small.

Rejection of the signal component (quadrature component) 90° out of phase with the wanted signal component can be very good in this demodulator.

Fig. 3b shows the so called high-level demodulator. The idealized circuit can be represented by a switch, opening and closing at moments prescribed by the reference subcarrier. The action of this switch is to clamp the signal at a fixed level every time it closes.

In this way the desired video signal is recovered by clamping at the signal tips. Quadrature information is rejected because signals 90° out of phase are clamped at the moments the signal is passing through zero, thus producing no video component. This demodulator can deliver a usable signal of 80 to 100 V_{pp}. In order to obtain this large output the opening angle of the switch should be 60-100 degrees. Complete quadrature rejection is only possible at infinitesimal opening angles. Then however the output is infinitesimal too.

In contrast with the transient distortion produced by quadrature cross talk referred to in the previous chapter, the incomplete quadrature rejection mentioned above produces large area phase (hue) errors. The high level demodulator is used in cheap receivers, where the demodulator directly drives the the kinescope electrode without further amplification.

For video outputs up to 10 V_{pp} the demodulator of fig. 3c is a very useful circuit. This circuit acts like a bilateral switch, and the phase errors due to imperfect quadrature rejection are in general smaller than those encountered in high-level demodulators if only the applied reference subcarrier amplitude is 3 to 5 times the applied chrominance signal. This condition precludes the use of this circuit for video outputs of more than 10 V_{pp}.

For $I-Q$ receivers, where matrixing is necessary, this balanced diode demodulator is a useful circuit. The necessary amplification of 10 to 20 times between the demodulators and the kinescope electrodes can easily be incorporated in a matrix circuit, e.g. a common cathode matrix circuit (fig. 6)[6]).

4. Reference subcarrier regenerators.

The reference subcarrier signal used to operate the synchronous demodulators should have a fixed phase relationship to the subcarrier used in the encoder. Phase synchronization is made possible by transmission of a 'burst' of 10 cycles of subcarrier on the backporch of the horizontal synchronization signal. A static phase synchronization error smaller than $\pm$ 5 degrees is permissible. Furthermore at low signal to noise ratios a phase noise of 10 to 15 degrees r.m.s. is tolerable.

This amount of phase noice is usually tolerated at

$$\frac{S}{N} = \frac{\text{r.m.s. burst}}{\text{r.m.s. noise}} \geqslant 0.25 \text{ in the composite signal}$$

with 5 Mc/s bandwidth.

Static errors result from a difference between the freerunning frequency of the undisturbed regenerator and the frequency of the impressed synchronization signal.

The static phase errors can be eliminated by providing a customer controlled phase shifter (hue control). Practical experience, however, shows that the layman generally finds it difficult to obtain the correct setting of the hue control; it appears that he is inclined to readjust the hue control in a different position for each item of the colour programme. This indicates that it might be wise to eliminate the customer-controlled hue control. In that case the regenerator should have

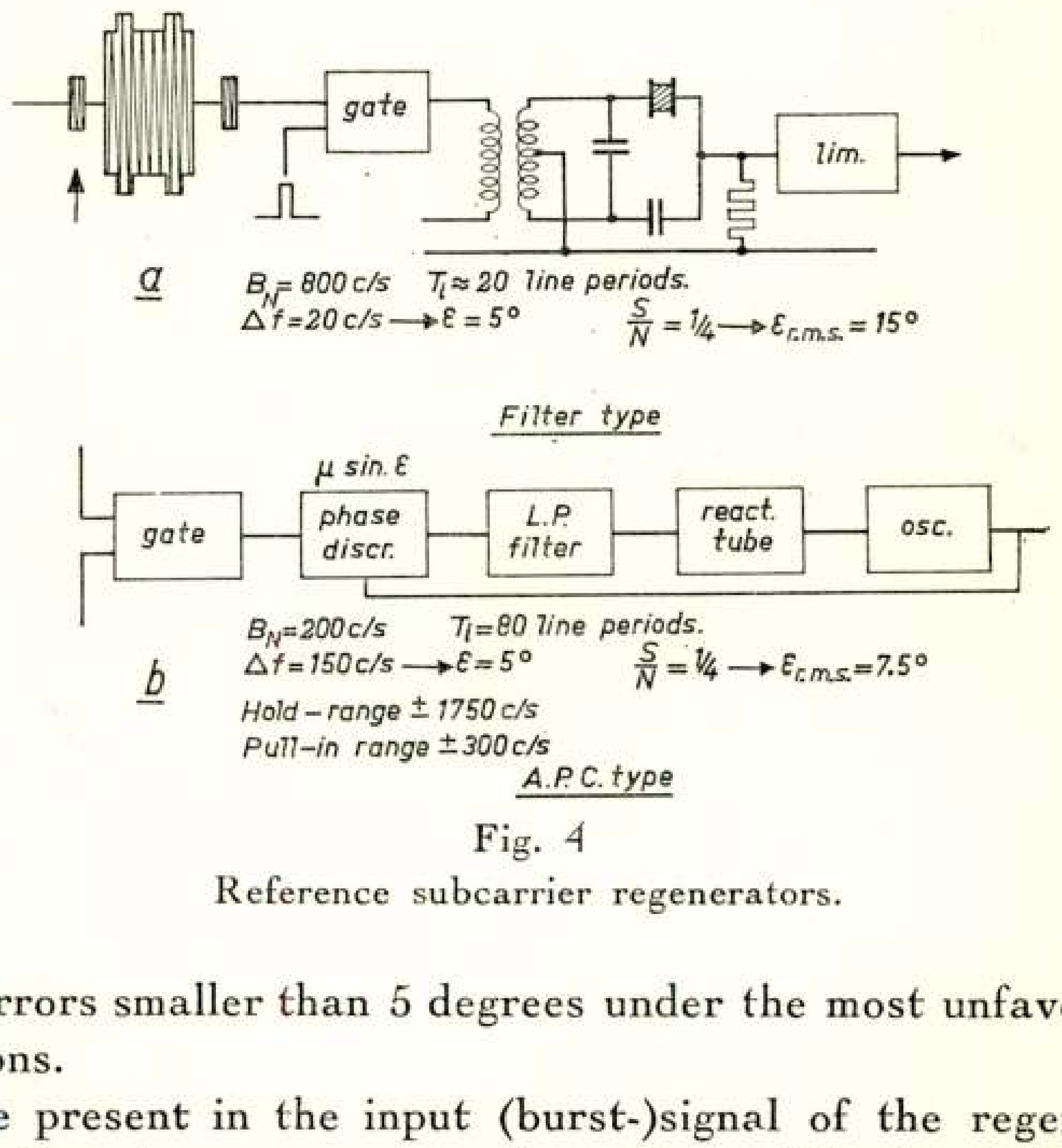

Fig. 4
Reference subcarrier regenerators.

static errors smaller than 5 degrees under the most unfavourable conditions.

Noise present in the input (burst-)signal of the regenerator diffuses the phase measurements made on this signal, but integration of the timing information present in successive bursts can be used to decrease the uncertainy [10]).

The overall capability of a regenerator to reject thermal noise is usually described by stating a virtual integration time or an equivalent noise bandwidth. From either figure the r.m.s. output phase noise can be computed for a given video signal to noise ratio. [10])

Two types of regenerators are indicated in fig. 4.

The circuit of fig. 4a consists essentially of a crystal filter and a limiter. Readily realizable circuit properties are given in the figure.

The noise performance is marginal. It can be improved by giving the circuit a higher Q-value, but this increases the static error, even for small values of the difference frequency. The static performance can be improved by incorporating an automatic phase control loop, but then the attractive simplicity of the circuit is gone.

A second drawback of this regenerator results from the direct application of the bursts to the crystal.

Not only is the fundamental mode of the crystal excited in this way but under certain circumstances spurious modes in the vicinity of the fundamental as well.

The result is a regenerated subcarrier, modulated in amplitude and phase by the difference frequency (or frequencies) of the fundamental and the spurious mode (or modes). This produces discoloured vertical stripes in the picture.

By contour grinding of the crystal the spurious modes can be suppressed[11]) but this will no doubt lead to a more expensive regenerator circuit.

The most flexible regenerator is the type of fig. 4b. Here the frequency of a crystal oscillator is adjusted by a reactance valve, which is controlled by the error information obtained from a phase comparison between the subcarrier output and the burst input voltages.

The noise performance and the static performance of this regenerator are not so rigidly coupled as in the type of fig. 4a. Enlarging the product (the loop gain) of the phase discriminator sensitivity and the reactance tube sensitivity improves the static performance. Lowering the low-pass filter cut-off frequency improves the noise performance.

A complication is introduced by the pull-in behaviour during initial synchronization. The pull-in range, denoting the range of difference frequencies where synchronization is ultimately reached, is affected by the loop gain and the filter cut-off. A usable compromise can normally be found by correctly shaping the filter amplitude-frequency response[10]).

An example is given in fig. 4b for the case where the phase discriminator delivers a voltage $E = \mu \sin \varepsilon$ and the reactance tube-oscillator combination has an output frequency

$$\omega = \omega_0 + \beta E .$$

The circuit diagram of the regenerator used in our receiver is drawn in fig. 5. Here the reactance tube-oscillator system obeys the linear law given above only over a range that is small compared to the maximum possible discriminator output.

Outside this small range the system is allowed to saturate (fig. 5b), a measure that gives some advantages with regard

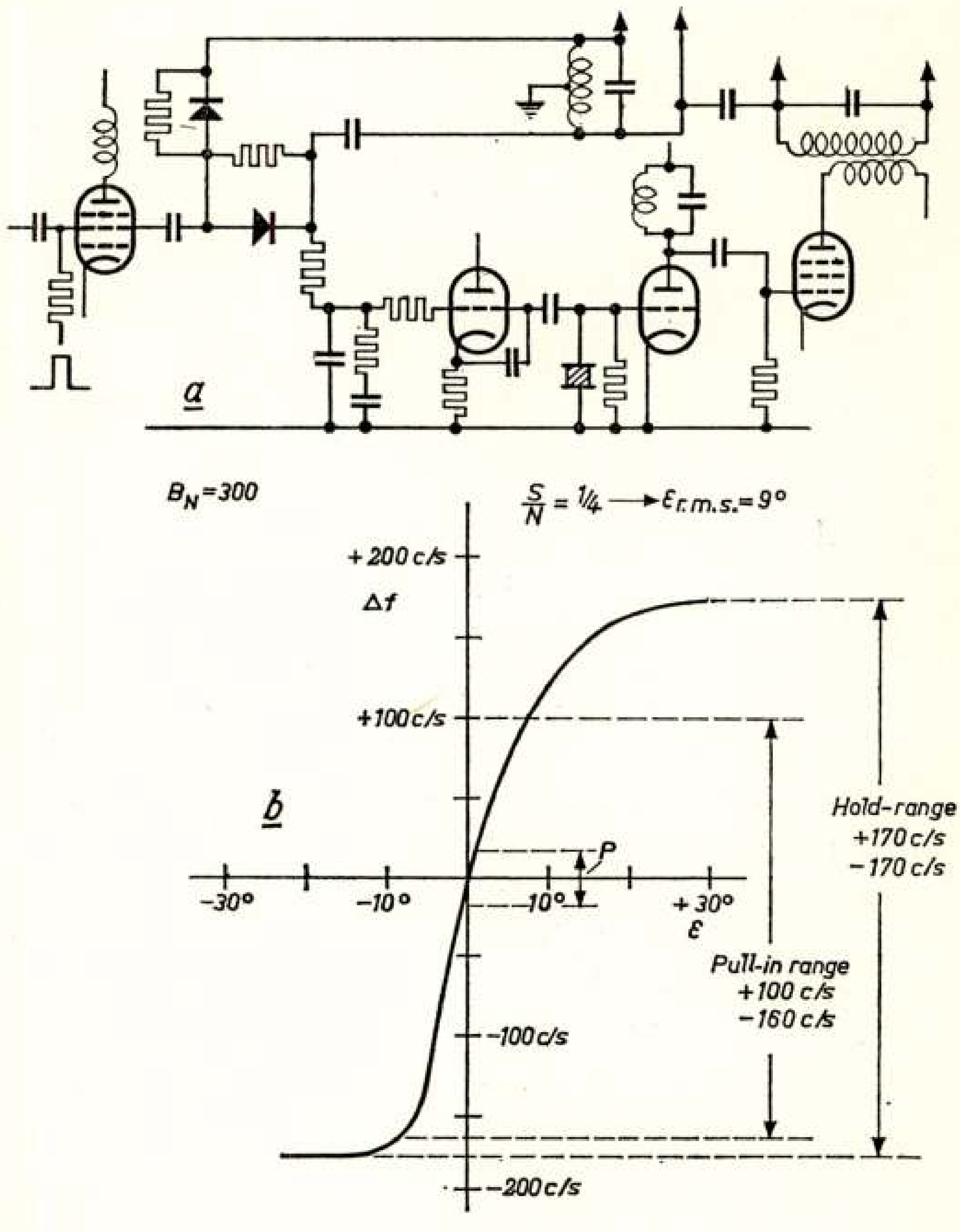

Fig. 5
Regenerator circuit.

to static phase stability. The hold range decreases considerably but this is of no consequence as we are only interested in the range from minus five to plus five degrees.

The range indicated by P in fig. 5b is the range of phase errors measured in a receiver under extreme operating conditions.

5. **Example of a receiver design.**

In fig. 6 a circuit diagram of the video luminance and chrominance circuits of our receiver is given omitting irrelevant details.

In the upper part the luminance signal is detected, amplified and delivered to the cathodes of the guns via a delay line.

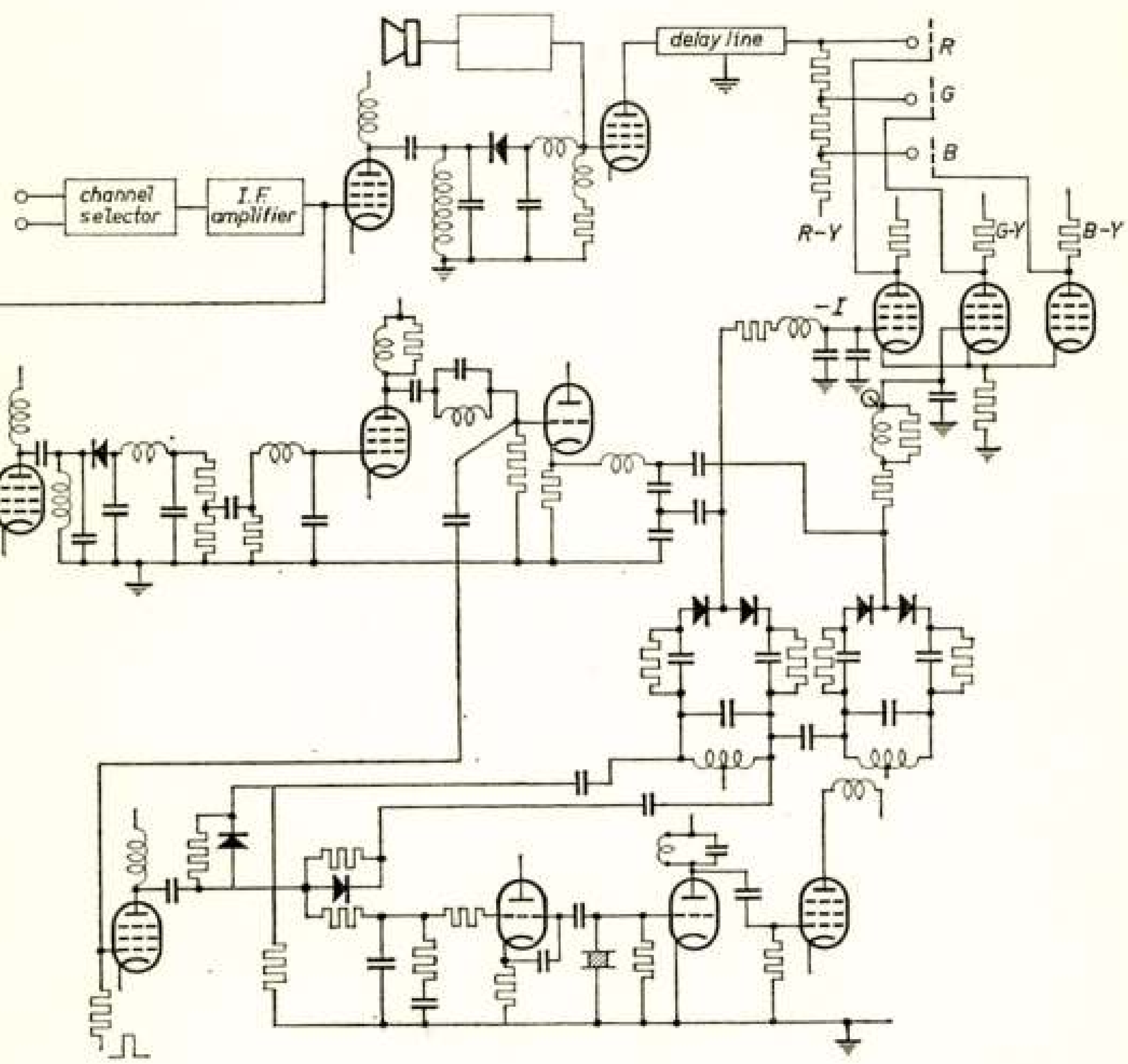

Fig. 6
Video and colour circuits.

The lower part of fig. 6 will be recognized as the regenerator circuit of fig. 5a. In the centre part the i.f. signal is detected. The following band-pass amplifier rejects the luminance signal components and delivers the chroma signal to the two synchronous demodulators working along the minus I and Q axes. The last stage is a common cathode matrixing amplifier where the minus I and Q signals are used to produce the colour difference signals $R-Y$, $G-Y$ and $B-Y$.

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Large screen colour television projectors

by T. Poorter *)

Summary of a lecture read before the Nederlands Radio Genootschap on November 20th 1958.

The information presented in this lecture has already been published elsewhere **). Therefore only a summary of the subject matter is given here.

A description is given of two-large screen colour television projectors which were constructed at the Philips Research Laboratories in Eindhoven.

In both projectors the colour picture is obtained by a simultaneous projection of three images, each in a primary colour (red, green, blue) on a single projection screen. The three primary images should coincide exactly on the projection screen.

The primary images are projected by three Schmidt optical systems each containing a 5" projection cathode ray tube provided with a phosphor coating that radiates light of the required primary colour.

In one of the projectors use is made of dichroic mirrors. As seen from the projection screen the three optical system are then virtually situated in the same place.

In this case the distortion resulting from projection is the same for all the three primary images. If the corresponding components used in the three optical systems for the formation of the primary images were identical, no superposition errors would appear on the projection screen.

The construction of this projector is rather delicate and a loss of light of 50% results from the use of the dichroic mirrors.

The other projector uses three Schmidt optical systems situated side by side. This gives rise to an optical trapezoidal distortion of the primary images projected by the outer pro-

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**) Philips Technical Review 19, 338-355, 1957/1958 (No. 12).

jectors. By predistorting the deflection currents of the respective projection tubes this distortion can be corrected.

The absence of dichroic mirrors and the use of straight optical systems result in a gain in highlight luminance, contrast ratio and resolution when compared to the first mentioned projector. On a screen of 7 m^2 a highlight luminance of 20 cd/m^2 and a resolution of 600 lines is obtained. The contrast ratio is about 35.

After the lecture a colour television demonstration was given in which the last mentioned projector was used. The programme consisted of: pictures of colour transparancies, which pictures were produced by means of a flying spot scanner; live scenes taken by a vidicon colour camera and a short piece of colour film taken by a vidicon film scanner.

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General energy relations for parametric amplifying devices

by S. Duinker *)

Summary

It is shown that the energy relations pertaining to parametric amplifying devices, as they have been derived by various authors, are a direct consequence of the invariance of the total-energy function of the parametric system under certain transformations. The theory is generalized so as to comprise arbitrary parametric systems. Some general properties of parametric systems, which can be deduced immediately from the energy relations, are discussed. A small number of typical examples are briefly treated to illustrate some fundamental principles following from the general theory.

1. Introduction.

The name of parametric amplifier is generally accepted to indicate a class of devices of very different physical nature but with the common property that the amplification results from the variation of some reactive circuit parameter. These devices contain, apart from non-linear reactances or time-dependent reactances, linear impedances and periodically varying sources of energy. They are part of the class of frequency-converting systems. A survey of the literature on the relations governing the exchange of energy taking place at the various frequencies in frequency-converting networks is given in ref. [1]).

Under special circumstances the frequency conversion in a parametric system can give rise to the occurrence of negative resistances, which may be used either for the generation of oscillations or for amplification. Recent proposals of Suhl [2]), Uhlir [3]) and others (see ref. [1])), to use parametric amplifiers at micro-wave frequencies, presumably with a low inherent noise level, have resulted in a revival of the interest in the theory of these devices.

Hitherto the theory was confined almost exclusively to spe-

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cial cases where only a few frequency components were taken into account. Notwithstanding the restrictions underlying such analyses, the mathematical calculations necessary for gaining an insight into the properties of the particular system considered have often been rather lengthy and, moreover, they do not lend themselves to generalization. The simple heuristic approach of Weiss [4]), which is based upon a quantum-mechanical energy-level model and which resembles very much a suggestion made by Hartley [5]) much earlier, also suffers from not being very useful for the purpose of generalization.

In recent years there have been various attempts to generalize the energy relations pertaining to parametric amplifying devices so as to comprise the entire range of frequencies occurring in the conversion process. In this connection mention should be made of the work of Manley and Rowe [6]) (who derived the energy relations for a single non-linear reactance connected to two large-signal generators simultaneously), of Duinker [1]) (who considered an arbitrary non-linear reactive network connected to a small-signal generator and a carrier source), and of Page [7]) (who studied general lossless systems in conjunction with a large number of carrier sources, unfortunately however, in an incorrect way).

When comparing the results obtained by the various authors one is struck by the fact that the energy relations assume a similar form in all cases, irrespective of the differences in complexity and physical arrangement of the parametric system. Thus one is led to the conjecture that these energy relations can be deduced in a more straightforward way from some fundamental principle.

It is the purpose of the present paper to demonstrate that the energy relations are a direct consequence of the invariance of the total-energy function of a general parametric system under certain transformations. The theory developed here is a generalization of the analysis of Manley and Rowe [6]). A few typical examples are briefly discussed to illustrate some fundamental principles following from the general theory.

2. **Specification of the system considered.**

In order to arrive at certain energy relations as our final result as quickly as possible, the analysis will first be confined to a system that, although being fairly general, is not the most general system satisfying these energy relations. The generalization of our results will be treated in Sec. 4. To facilitate the reasoning in our considerations, which are on the basis of generalized dynamics, we shall introduce some notions which are borrowed from network theory, such as: the notion of "source of energy", and the notion of "port", to indicate those parts of the system where energy (resulting from some conversion process which is of no interest) appears in the dynamical structure for the first time in a form which has to be taken into account, and to localize the places where exchange of energy between parts of the system takes place, respectively.

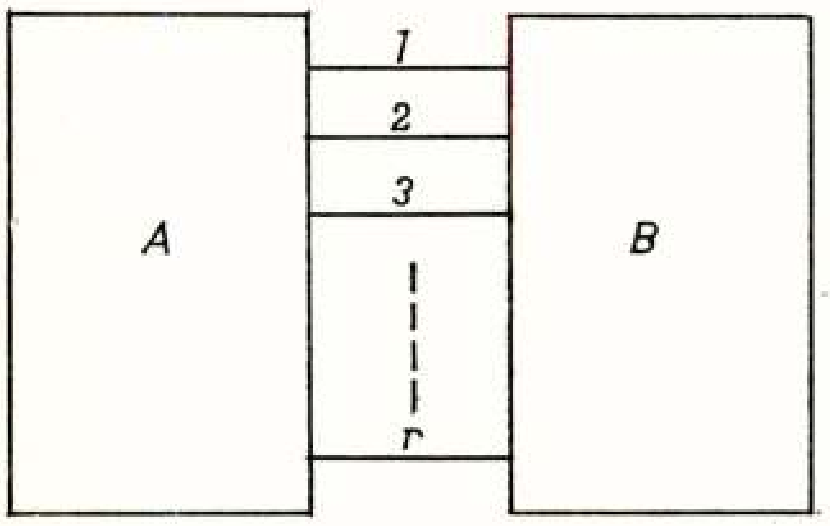

Fig. 1.
Parametric system comprising a conservative non-linear r-port A, and a linear dissipative (but not necessarily purely dissipative) r-port B, which additionally contains an arbitrary number of periodically varying sources of energy with basis frequencies $\omega_1, \ldots, \omega_k$.

Consider the system in Fig. 1 which comprises two subsystems, A and B, exchanging energy through r ports. One of the r-ports, A, contains conservative non-linear elements, capable of storing energy. The other r-port, B, corresponds to a dissipative system comprising periodically varying sources of energy, k in number, with mutually incommensurate positive frequencies, $\omega_1, \ldots, \omega_k$. The r-port A is understood to form a connected system; r-port B, however, may consist of isolated parts. Although this is not strictly necessary, as we shall see in Sec. 4, we shall assume the r-port B to contain linear dissipative and possibly also linear conservative elements.

The behaviour of the conservative r-port A (whether linear or not) can be described by the set of Lagrangian equations:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}_s} - \frac{\partial L}{\partial x_s} = y_s\,, \quad s = 1, \ldots, m, \tag{1}$$

when it is assumed to possess m degrees of freedom [8]).

Let the ports be numbered $1, \ldots, r$, then the indices $s = r + 1, \ldots, m$ correspond to "internal" degrees of freedom. In (1), y_s denotes the generalized force impressed at the s th degree of freedom and hence $y_s = 0$ for $s = r + 1, \ldots, m$.

The physical nature of the coordinates x_s and their corresponding velocity $\dot{x}_s = dx_s/dt$ depends upon the character of the forces acting on the ports, which, in their turn, depend on the character of the sources of energy present in the "external" r-port B. A specification of the nature of these sources of energy is not required for the following considerations; only their dynamical behaviour, i.e., their time-dependency, will be of interest. Nevertheless it might be illustrative to indicate a few possibilities.

If y_s represents a mechanical force, then x_s and $\dot{x}_s$ stand for "true" coordinates and their corresponding velocity, respectively. If, in the electrical case, a voltage source is operating in the s th degree of freedom, the generalized coordinate x_s corresponds to an electric charge q_s and hence $\dot{x}_s$ to a current $i_s = \dot{q}_s$. On the other hand, when y_s represents an injected current, $\dot{x}_s$ and and x_s stand for the voltage, $e_s = \dot{\varphi}_s$, and its time-integral, the flux φ_s, respectively.

The function

$$L = L\,(x_1, \ldots, x_m;\, \dot{x}_1, \ldots, \dot{x}_m) \tag{2}$$

represents the Lagrangian or the kinetic potential of r-port A. Since the sub-system A is supposed to be non-linear, L is a function which is of higher than the second degree in the variables. A further specification of L is not required at this place.

As a result of the periodic sources of energy present in subsystem B, a steady state for the distribution of the quantities y_s, x_s and $\dot{x}_s$ will exist in the entire system, A and B together, after the transients have died out as a result of the dissipation assumed to take place in B. This steady state is characterized by the presence of new frequencies which are derived from the basis frequencies ω_j $(j = 1, \ldots, k)$ through (inter-)modulation and which are therefore linearly related to them. Hence any frequency that occurs in the system can be written in the form:

$$\omega_{n_1, \ldots, n_k} = \sum_{j=1}^{k} n_j\, \omega_j \;,\; n_j = \ldots, -1, 0, +1, \ldots, \tag{3}$$

in which the ω_j represent the basis frequencies (viz., $\omega_1 = \omega_{1,0,0,\ldots,0}$, $\omega_2 = \omega_{0,1,0,\ldots,0}$, $\ldots$, $\omega_k = \omega_{0,0,0,\ldots,1}$), whereas the other frequencies are derived ones.

After introducing for convenience the new dimensionless variables

$$\Omega_j = \omega_j t \ , \ j = 1, \dots, k \ ,$$

and

$$\Omega_{n_1, \dots, n_k} = \omega_{n_1, \dots, n_k} \, t \ ,$$

we can write in the steady state for the quantity y_s corresponding to a port s ($s = 1, \dots, r$) the following multiple Fourier expansion:

$$y_s = \sum_{n_1, \dots, n_k = -\infty} \dots \sum^{+\infty} {}^s Y_{n_1, \dots, n_k} \, exp \, (i \, \Omega_{n_1, \dots, n_k}) \ . \tag{4}$$

The complex coefficients are determined by:

$${}^s Y_{n_1, \dots, n_k} = (2 \pi)^{-k} \int_0^{2\pi} \dots \int_0^{2\pi} d\,\Omega_1 \dots d\,\Omega_k y_s \, exp \, (-i \, \Omega_{n_1, \dots, n_k}) \tag{5}$$

and since y_s represents a real-valued physical quantity, the coefficients satisfy the equality:

$${}^s Y_{n_1, \dots, n_k} = \tfrac{1}{2} \, {}^s y_{n_1, \dots, n_k} \, exp \, (i \, {}^s \varphi_{n_1, \dots, n_k}) = {}^s Y^*_{-n_1, \dots, -n_k} \ . \tag{6}$$

In (6) the asterisk denotes the complex conjugate and ${}^s y_{n_1, \dots, n_k}$ and ${}^s \varphi_{n_1, \dots, n_k}$ represent the amplitude and the phase angle, respectively, of the component with frequency $\omega_{n_1, \dots, n_k}$ at port s.

For the coordinate x_s that determines the s th degree of freedom ($s = 1, \dots, m$) we have in a similar way:

$$x_s = \sum_{n_1, \dots, n_k = -\infty} \dots \sum^{+\infty} {}^s X_{n_1, \dots, n_k} \, exp \, (i \, \Omega_{n_1, \dots, n_k}) \ , \tag{7}$$

from which we obtain the corresponding velocity by differentiation with respect to time:

$$\dot{x}_s = \sum_{n_1, \dots, n_k = -\infty} \dots \sum^{+\infty} i \, \omega_{n_1, \dots, n_k} {}^s X_{n_1, \dots, n_k} \, exp \, (i \, \Omega_{n_1, \dots, n_k}) \ . \tag{8}$$

The relationships between the complex coefficients ${}^s X_{n_1, \dots, n_k}$ and the corresponding amplitudes ${}^s x_{n_1, \dots, n_k}$ and phase angles ${}^s \psi_{n_1, \dots, n_k}$ are given by

$$\begin{aligned} i \, \omega_{n_1, \dots, n_k} {}^s X_{n_1, \dots, n_k} &= \tfrac{1}{2} \, {}^s x_{n_1, \dots, n_k} \, exp \, (i \, \psi_{n_1, \dots, n_k}) \\ &= i \, \omega_{n_1, \dots, n_k} {}^s X^*_{-n_1, \dots, -n_k} \ . \end{aligned} \tag{9}$$

It should be remarked here that the expressions (7) and (8) imply a particular (but otherwise arbitrary) choice of the zero point of time. If we consider the same expressions for the case that, while leaving everything else unchanged, the phase of the energy source with basis frequency ω_j, reckoned with respect to the same zero point of time, is shifted over an angle Φ_j (i.e., a transformation of any argument Ω_j to $\Omega_j + \Phi_j$, and hence, of any argument $\Omega_{n_1,\ldots,n_k}$ to $\Omega_{n_1,\ldots,n_k} + n_j\,\Phi_j$), then the complex coefficients $X_{n_1,\ldots,n_k}$ in the expressions (7) and (8) are subjected to the transformation:

$$X_{n_1,\ldots,n_k} \rightarrow exp\,(i\,n_j\,\Phi_j)\,X_{n_1,\ldots,n_k}\,.$$

Hence, if Φ_j is increased from zero to 2π the value of the quantities x_s and $\dot{x}_s$ will remain unchanged.

In the next section we will examine the behaviour of the system at a fixed instant (*e.g.*, $t = \tau$) and for fixed values of the basis frequencies ω_j, but for virtual variations of the phase angles Φ_j of the individual basis frequency sources. In that case we can consider x_s and $\dot{x}_s$ as functions of the indopendent variables Φ_j, or more conveniently, as functions of the independent variables Ω_j $(j = 1,\ldots,k)$, what comes to the same thing if we keep the rest of the parameters (ω_j and t) unchanged.

3. **Derivation of the energy relations.**

At any instant the power that is flowing into the conservative sub-system A through the port s is, in virtue of eqs (1) and (2):

$$\begin{aligned}\dot{x}_s\,y_s &= \dot{x}_s\,\frac{d}{dt}\,\frac{\partial L}{\partial \dot{x}_s} - \dot{x}_s\,\frac{\partial L}{\partial x_s}\\ &= \frac{d}{dt}\left(\dot{x}_s\,\frac{\partial L}{\partial \dot{x}_s}\right) - \ddot{x}_s\,\frac{\partial L}{\partial \dot{x}_s} - \dot{x}_s\,\frac{\partial L}{\partial x_s}\,.\end{aligned}$$

The total instantaneous power flowing into sub-system A is, therefore:

$$\sum_{s=1}^{m} \dot{x}_s\,y_s = \sum_{s=1}^{m} \frac{d}{dt}\left(\dot{x}_s\,\frac{\partial L}{\partial \dot{x}_s} - L\right) = \frac{dH}{dt},$$

if H denotes the Hamiltonian of A which corresponds to the total energy stored in A. Note that $y_s = 0$ for $s = r + 1,\ldots,m$. Obviously, on account of (2), H can be characterized as follows:

$$H = \sum_{s=1}^{m}\left(\dot{x}_s\,\frac{\partial L}{\partial \dot{x}_s} - L\right) = H\,(x_1,\ldots,x_m;\ \dot{x}_1,\ldots,\dot{x}_m). \quad (10)$$

The conversatism assumed for the r-port A implies that after a full cycle of variations of state of coordinates and velocities, the r-port A has neither absorbed nor delivered energy. In other words, H is a uniquely defined function of its variables x_s and $\dot{x}_s$ $(s = 1, \ldots, m)$, possibly apart from a non-essential additive constant which can be removed by defining $H(0, \ldots, 0; 0, \ldots, 0) = 0$. Under the steady-state conditions considered in the present case these variables are given by the multiple Fourier series (7) and (8).

If we examine the total-energy function H at a fixed instant and for fixed values of the basis frequencies, but for other values of the phase angles of the various basis frequency sources (reckoned with respect of an arbitrarily chosen, but fixed zero point of time), as was discussed at the end of the previous section, then we may write instead of (10):

$$H = H(\Omega_1, \ldots, \Omega_k)\,. \tag{11}$$

We can now consider the effect on H of virtual variations of the independent parameters Ω_j. From the reasoning presented before it follows that the quantities x_s and $\dot{x}_s$ will not change (i.e., will pass through a full cycle), if any one of the new variables Ω_j is increased by an amount of 2π. Consequently, the energy function (11) will also be invariant under such changes. Therefore, if the differential expression

$$dH = \sum_{j=1}^{k} \frac{\partial H}{\partial \Omega_j}\, d\Omega_j \tag{12}$$

is integrated between the limits 0 and 2π, each term on the right will be equal to zero, i.e.,

$$\int_0^{2\pi} \frac{\partial H}{\partial \Omega_j}\, d\Omega_j = 0\,,\; j = 1, \ldots, k\,. \tag{13}$$

Expression (12) can be written in another form, namely:

$$\begin{aligned} dH &= \sum_{s=1}^{m} \dot{x}_s y_s\, dt = \sum_{s=1}^{m} y_s\, dx_s \\ &= \sum_{s=1}^{m} y_s \left(\sum_{j=1}^{k} \frac{\partial x_s}{\partial \Omega_j}\, d\Omega_j \right). \end{aligned} \tag{14}$$

After identification of corresponding coefficients in (12) and (14) we obtain instead of (13):

$$\sum_{s=1}^{m} \int_0^{2\pi} y_s \frac{\partial x_s}{\partial \Omega_j}\, d\Omega_j = 0\,,\; j = 1, \ldots, k\,. \tag{15}$$

The partial derivative with respect to Ω_j of the expression (7) for x_s is:

$$\frac{\partial x_s}{\partial \Omega_j} = \sum_{n_1,\ldots,n_k=-\infty}^{+\infty} i n_j\, {}^sX_{n_1,\ldots,n_k} \exp(i\,\Omega_{n_1,\ldots,n_k})$$

$$= -\sum_{n_1,\ldots,n_k=-\infty}^{+\infty} i n_j\, {}^sX^*_{n_1,\ldots,n_k} \exp(-i\,\Omega_{n_1,\ldots,n_k}),$$

since

$$\frac{\partial\, \Omega_{n_1,\ldots,n_k}}{\partial\, \Omega_j} = n_j\,,$$

so that (15) assumes the following form:

$$\sum_{s=1}^{m} \sum_{n_1,\ldots,n_k=-\infty}^{+\infty} \int_0^{2\pi} d\,\Omega_j y_s \exp(-i\,\Omega_{n_1,\ldots,n_k}) i n_j\, {}^sX^*_{n_1,\ldots,n_k} = 0,\ j = 1,\ldots,k. \quad (16)$$

On the other hand, by multiplying both sides of eq. (5) by $i n_j\, {}^sX^*_{n_1,\ldots,n_k}$ and summing $n_1,\ldots,n_k$ from $-\infty$ to $+\infty$, and s from 1 to m, we have:

$$i \sum_{s=1}^{m} \sum_{n_1,\ldots,n_k=-\infty}^{+\infty} n_j\, {}^sX^*_{n_1,\ldots,n_k}\, {}^sY_{n_1,\ldots,n_k} =$$

$$(2\pi)^{-k} \sum_{s=1}^{m} \sum_{n_1,\ldots,n_k=-\infty}^{+\infty} \int_0^{2\pi} \ldots \int_0^{2\pi} d\,\Omega_1 \ldots d\,\Omega_k y_s \exp(-i\,\Omega_{n_1,\ldots,n_k})\, i n_j\, {}^sX^*_{n_1,\ldots,n_k}.$$

This latter expression is equal to zero on account of (16), so that k relations are found ($j = 1,\ldots,k$):

$$i \sum_{s=1}^{m} \sum_{n_1,\ldots,n_k=-\infty}^{+\infty} n_j\, {}^sX^*_{n_1,\ldots,n_k}\, {}^sY_{n_1,\ldots,n_k} = 0, \quad (17)$$

namely, one for each basis frequency.

Relations (17) can be written in a more convenient form if we realize that the total average power, $P_{n_1,\ldots,n_k}$, flowing into the r-port A at a frequency $\omega_{n_1,\ldots,n_k}$, amounts to:

$$P_{n_1,\ldots,n_k} = i\,\omega_{n_1,\ldots,n_k} \sum_{s=1}^{m} ({}^sX^*_{n_1,\ldots,n_k}\, {}^sY_{n_1,\ldots,n_k} + {}^sX_{n_1,\ldots,n_k}\, {}^sY^*_{n_1,\ldots,n_k}) =$$

$$i\,\omega_{n_1,\ldots,n_k} \sum_{s=1}^{m} ({}^sX^*_{n_1,\ldots,n_k}\, {}^sY_{n_1,\ldots,n_k} + {}^sX^*_{-n_1,\ldots,-n_k}\, {}^sY_{-n_1,\ldots,-n_k}) = P_{-n_1,\ldots,-n_k}, \quad (18)$$

if use is made of (6) and (9). Furthermore, on account of (3),

$$n_j\,\omega^{-1}_{n_1,\ldots,n_k} = -n_j\,\omega^{-1}_{-n_1,\ldots,-n_k}. \quad (19)$$

Hence, after arranging the terms on the left of (17) in pairs with opposite indices, the substitution of (18) and (19) yields the desired expressions:

$$\sum_{n_1=-\infty}^{+\infty} \ldots \sum_{n_j=1}^{+\infty} \ldots \sum_{n_k=-\infty}^{+\infty} n_j \, \omega_{n_1,\ldots,n_k}^{-1} P_{n_1,\ldots,n_k} = 0, j = 1, \ldots, k. \qquad (20)$$

If sub-system A is a one-port ($r = 1$) and sub-system B contains two basis-frequency sources ($k = 2$), relations (20) reduce to:

$$\sum_{n_1=1}^{+\infty} \sum_{n_2=-\infty}^{+\infty} \frac{n_1 \, P_{n_1, n_2}}{n_1 \omega_1 + n_2 \omega_2} = 0 \text{ and } \sum_{n_2=1}^{+\infty} \sum_{n_1=-\infty}^{+\infty} \frac{n_2 \, P_{n_1, n_2}}{n_1 \omega_1 + n_2 \omega_2} = 0,$$

as was found by Manley and Rowe [6]).

If sub-system A is a general reactive non-linear multi-port and sub-system B contains two basis-frequency sources ($k = 2$), one of which (say ω_1) is a small-signal source, so that we only have to take into account $n_1 = -1, 0, +1$, then the relations (20) assume the form:

$$\sum_{n_2=-\infty}^{+\infty} \frac{P_{1,n_2}}{\omega_1 + n_2 \omega_2} = 0 \text{ and } \sum_{n_2=1}^{+\infty} \sum_{n_1=-1}^{+1} \frac{n_2 \, P_{n_1, n_2}}{n_1 \omega_1 + n_2 \omega_2} = 0,$$

a result that has previously been obtained in a different way by Duinker [1]).

4. Generalization of the energy relations.

For the derivation of the energy relations (20), certain restrictions, which were formulated in Sec. 2, were assumed to apply to the system. These restrictions will now be considered more closely and we will try to remove some of them.

An essential postulate underlying our considerations is that ultimately a steady state, defined by a set of linearly related frequencies, will result from the influence of periodic sources of energy, so that it has sense to speak of average power quantities associated with a particular frequency component. Therefore, it is necessary to assume dissipation to take place in the external sub-system, the r-port B. However, for the derivation of the energy relations no additional requirements as to the dissipation have been used so that there is no reason to exclude sub-systems B which are *dissipative* and *non-linear*.

If r-port B is assumed to be non-linear and dissipative, then the process of frequency conversion (i.e., the creation of new frequencies linearly related to the basis frequencies ω_j as expressed by eq. (3)) is no longer taking place solely in r-port A, but also in r-port B. At first sight it looks as if the reasoning in Sec. 3, and hence the energy relations, still hold good if A is a linear conservative sub-system, and B an active non-linear sub-system. However, a closer examination reveals that then the average power associated with *each* frequency component is equal to zero and hence that the energy relations have lost their importance. Additional conditions imposed on the Lagrangian for r-port A will be necessary to ensure that A is actively taking part in the process of frequency conversion. These conditions will be mentioned in the sequel.

The assumption mentioned in Sec. 2 that r-port A corresponds to a conservative system implies that A is not only a *holonomic* system (i.e., a system that is completely specified by independent coordinates which are equal in number to the number of degrees of freedom), but also a *scleronomic* system (i.e., a dynamical system for which the Lagrangian L does not depend explicitly on time, as is expressed by (2)). In classical linear dynamics, L is known to be equal to the difference of the kinetic energy (which depends on the velocities) and the potential energy (which depends on the coordinates). Cherry [8]) has shown that, for the Lagrangian formalism to hold for non-linear electrical systems, some other quantity, the kinetic co-energy, has to be introduced in L instead of the kinetic energy. In the general case more complicated forms of the Lagrangian will hold, for instance, in relativistic dynamics where the masses depend non-linearly on the velocities. A specification of the Lagrangian as to its form is not required for our considerations. In any case where the system remains *holonomic*, *scleronomic* and *non-linear*, the Lagrangian L will be a uniquely defined function of the coordinates and the velocities and so will be the Hamiltonian H, and hence the energy relations will hold.

The requirement of non-linearity, which is necessary in the scleronomic case to ensure that the system takes part in the process of frequency conversion, implies that at least one of the equalities

$$\frac{\partial^2 L}{\partial x_s\, \partial x_r} = \text{const.}, \frac{\partial^2 L}{\partial x_s\, \partial \dot{x}_r} = \text{const.}, \frac{\partial^2 L}{\partial \dot{x}_s \partial \dot{x}_r} = \text{const.}; s, r = 1, \ldots, m, \quad (21)$$

is *not* satisfied. Systems belonging to this category are sometimes called *auto-parametric* systems to express the fact that the reactions of the system upon external forces are functions of the coordinates and of the velocities. Examples of these are purely reactive non-linear networks such as magnetic [9]) and dielectric [10]) [11]) modulators.

We shall now investigate whether the requirement of conservatism for r-port A is really necessary. What is actually used in the derivation of the energy relations is not the conservatism of the system but rather the fact that, under the steady-state conditions postulated in the foregoing, the total-energy function H in (11) remains invariant under a transformation of the variables $\Omega_j \rightarrow \Omega_j + 2\pi$. This latter requirement not necessarily implies that the system should be conservative. If the system, apart from being *holonomic*, is *rheonomic* (i.e., possesses a Lagrangian which depends explicitly on time) then it is still possible for the total energy to be a periodic function of the variables Ω_j, although the system ceases to be conservative. This will now be discussed.

Consider a rheonomic Lagrangian function

$$L(x_1, \ldots, x_m;\ \dot{x}_1, \ldots, \dot{x}_m;\ t),$$

then we find by a reasoning similar to the one given in Sec. 3, that the total instantaneous power flowing into r-port A is given by:

$$\sum_{s=1}^{m} \dot{x}_s y_s = \frac{d}{dt}\left(\sum_{s=1}^{m} \dot{x}_s \frac{\partial L}{\partial \dot{x}_s} - L\right) - \frac{\partial L}{\partial t}.$$

Now, if $\partial L/\partial t$ can be expressed as a multiple Fourier series with frequencies which also are linear combinations of the basis frequencies ω_j, then it is again possible to define a function:

$$H' = \sum_{s=1}^{m}\left(\dot{x}_s \frac{\partial L}{\partial \dot{x}_s} - L\right) - \int \frac{\partial L}{\partial t}\, dt$$

which, under the steady-state conditions assumed, can be written in the form

$$H' = H'(\Omega_1, \ldots, \Omega_k).$$

This form will again be invariant under the transformation $\Omega_j \rightarrow \Omega_j + 2\pi$. Applying a similar reasoning as was presented before it is shown that the energy relations hold in this case too.

It is easily verified that even if the conditions (21) are all identically satisfied (i.e., if the system is linear) but $\partial L/\partial t$ is a multiple Fourier series of the form just discussed, the energy relations remain valid. Systems of this latter kind are sometimes referred to as *hetero-parametric* systems, to express the fact that linear system parameters (for instance, a capacitance for which the Lagrangian is $-2^{-1} f(t)\, x^2$) are made dependent on time by some external agent [12]). In accordance with the formalism adhered to throughout this paper, these external agents must be considered as sources of energy situated in the external sub-system B and acting upon sub-system A through its ports.

It should be noted that the foregoing analysis is not restricted to *lumped* parametric systems but that it also applies to *distributed* parametric systems, continuous media, etc., the difference being merely that scalar total energy density functions H' are involved which are expressed by an integral rather than by a polynomial form. As a special case of this latter type the propagating systems in which distributed reactances which vary periodically in time and space, may be regarded. Examples of these are the travelling-wave amplifiers using transmission lines embedded in a ferromagnetic medium [13]) [14]) [15]) or an electron beam [16]) as the time-dependent reactance. Of course, when extending the analysis to these more complicated systems the interpretation of an average power quantity $P_{n_1,\ldots,n_k}$ has to be modified so as to represent the average power associated with a frequency $\omega_{n_1,\ldots,n_k}$ and entering through a surface enclosing sub-system A. It depends entirely on the nature of the system whether or not a model with "ports" at which energy exchange with the exterior, sub-system B, takes place exclusively, is still possible.

In our discussion of the validity of the energy relations for holonomic systems we have so far overlooked one important aspect, namely, the proviso we made that r-port A should be *energic*, i.e., capable of storing energy. For purely non-energic holonomic systems (i.e., systems for which the total instantaneous power $\Sigma \dot{x}_s y_s$ flowing into them is equal to zero [17])) the total stored energy, or the Hamiltonian, satisfies

$$H = \sum_{s=1}^{m} \dot{x}_s \frac{\partial L}{\partial \dot{x}_s} - L = 0\,.$$

It remains a matter of choice whether one is inclined to call a non-energic system conservative, since there is no physical quantity that has to be conserved. If a non-energic system is characterized by a non-zero Lagrangian, the energy relations still hold true. An example of such a scleronomic non-energic non-linear system is supplied by the class of systems called ideal traditors[18]), which is defined by a Lagrangian of the form $L = A\, x_1 x_2 \ldots x_{m-1} \dot{x}_m$, where A represents a constant.

However, for non-energic systems which cannot be defined by a Lagrangian the energy relations no longer hold. To this category belongs the class of ideal transponators[19]) which includes ideal transformers and ideal gyrators with periodic transformation ratios and gyration resistances, respectively, periodically-operated ideal switches (switching modulators), ideal diodes, etc.

As a last step we shall investigate whether the requirement for the system to be holonomic is essential for the validity of the energy relations. If r-port A is *non-holonomic* the number of degrees of freedom is smaller than the number of independent coordinates, owing to the existence of auxiliary conditions between the coordinates.

So long as these relations are integrable, it can be shown[20]) by application of the Lagrangian method of undetermined multipliers that the conditions are dynamically equivalent to monogenic forces, i.e., forces derivable from a single scalar potential function, which is a function of the coordinates, the velocities and the time in the most general case. Impressed forces of the monogenic type can be absorbed into the Lagrangian so that the system remains holonomic.

Therefore, we have only to consider the case that the auxiliary conditions appear in the form of non-integrable relations between the differentials of the coordinates. Non-holonomic auxiliary conditions are maintained by forces which cannot be derived from a potential function. These forces, called *polygenic* forces, are again furnished by the method of undetermined multipliers but have to be considered as external forces. Such forces were included in the reasoning presented earlier in this paper so long as they fulfil the requirements set forth as to their multiple-periodic time-dependency.

The results obtained in Secs 2, 3 and 4 can be summarized in the following

Theorem:

In a dynamic system of m degrees of freedom, that in the steady state is exchanging energy through its ports with a dissipative (but not necessarily purely dissipative) external system comprising k periodically varying sources of energy with mutually incommensurate positive basis frequencies $\omega_1, \ldots, \omega_k$ and that is characterized by a non-zero Lagrangian function $L(x_1, \ldots, x_m; \dot{x}_1, \ldots, \dot{x}_m; t)$ and hence by a Hamiltonian function

$$H = \sum_{s=1}^{m} \dot{x}_s \frac{\partial L}{\partial \dot{x}_s} - L, \tag{10}$$

with the additional requirement that at least one of the quantities

$$\frac{\partial^2 L}{\partial x_s \, \partial x_r}, \; \frac{\partial^2 L}{\partial x_s \, \partial \dot{x}_r}, \; \frac{\partial^2 L}{\partial \dot{x}_s \, \partial \dot{x}_r} \; (s, r = 1, \ldots, m), \tag{21}$$

should not be equal to a non-zero constant, unless

$$\frac{\partial L}{\partial t} \neq 0,$$

the total average power $P_{n_1, \ldots, n_k}$ flowing into the system at the various frequencies

$$\omega_{n_1, \ldots, n_k} = \sum_{j=1}^{k} n_j \, \omega_j \; (n_j = \ldots, -1, 0, +1, \ldots), \tag{3}$$

obeys the relations

$$\sum_{n_1=-\infty}^{+\infty} \ldots \sum_{n_j=1}^{+\infty} \ldots \sum_{n_k=-\infty}^{+\infty} n_j \, \omega_{n_1, \ldots, n_k}^{-1} P_{n_1, \ldots, n_k} = 0 \; (j = 1, \ldots, k). \tag{20}$$

The theorem applies to any dynamical system that is characterized by a kinetic potential function $L(x_1, \ldots, x_m; \dot{x}_1, \ldots, \dot{x}_m; t)$. Physically, the nature of the generalized coordinates x_s could correspond to a charge, a flux, a spatial displacement, an angular rotation, etc., depending upon the nature of the impressed forces being, respectively, a voltage, a current, a mechanical force, a torque. Of course, general dynamical systems will be defined by coordinates which are not all of the same nature, e.g., electromechanical systems for which the coordinates are charges and displacements.

5. Discussion of the energy relations.

We shall now consider the power relations (20) more closely and try to draw some general conclusions from them.

Firstly, it should be noted that in each of the expressions (20) there is one set of terms that corresponds to one particular basis frequency. For instance, for the jth expression these are the terms

$$\sum_{n_j=1}^{\infty} \frac{n_j\, P_{n_j}}{n_j\, \omega_j} = \sum_{n_j=1}^{\infty} \frac{P_{n_j}}{\omega_j},$$

if we omit for shortness all suffixes n_i equal to zero except n_j.

Now we can rewrite the expressions (20) in the following form, after multiplying each expression by the corresponding basis frequency,

$$\sum_{n_j=1}^{\infty} P_{n_j} = \sum_{n_1=-\infty}^{+\infty}{}' \ldots \sum_{n_j=1}^{+\infty} \ldots \sum_{n_k=-\infty}^{-\infty}{}' \frac{n_j\, \omega_j}{\omega_{n_1,\ldots,n_k}} \quad P'_{n_1,\ldots,n_k},\ j=1,\ldots,k. \quad (22)$$

In (22), the prime at all summation signs but the one corresponding to n_j denotes that the terms on the right for which all suffixes n_i but n_j are simultaneously equal to zero must be deleted. The quantities

$$P'_{n_1,\ldots,n_k} = -P_{n_1,\ldots,n_k}$$

represent the total average power associated with a derived frequency $\omega_{n_1,\ldots,n_k}$, flowing *out* of sub-system A. It is to be noted that in each of the eqs (22) only those derived frequencies appear that are related to the respective basis frequencies; derived frequencies that are related exclusively to two or more of the other basis frequencies do not occur.

Expressions (22) can be interpreted as showing the manner in which the total average power delivered by the energy source with basis frequency ω_j, viz. ΣP_{n_j}, is distributed over the energy sinks or loads present at the various derived frequencies $\omega_{n_1,\ldots,n_k}$. In other words, they show to what extent an energy source, generating a basis frequency ω_j, contributes to the total average power dissipated at a particular derived frequency $\omega_{n_1,\ldots,n_k}$, namely, in the ratio of the corresponding frequencies, i.e., $n_j\omega_j/\omega_{n_1,\ldots,n_k}$.

A direct consequence of the latter conclusion is that the sum of all the average power quantities is equal to zero, i.e.,

$$\sum_{n_1=-\infty}^{+\infty} \ldots \sum_{n_j=0}^{+\infty}{}' \ldots \sum_{n_k=-\infty}^{+\infty} P_{n_1,\ldots,n_k} = 0, \quad (23)$$

so that *average* power is conserved for the parametric subsystem A under the assumed steady-state conditions. Since the system contains energy-storing elements the total sum of the *instantaneous* power quantities will in general not be equal to zero. In (23), the prime at the summation sign for the index n_j indicates that for each pair of terms with opposite frequencies occurring when n_j is equal to zero, only one term, for instance the one corresponding to the positive frequency, has to be taken into account.

Expression (23) implies that some of the power quantities (18) must be positive (indicating that power is flowing into the system as will usually be the case at a basis frequency although exceptions to this are possible as we shall see later), whereas others must be negative (indicating that power is extracted from the system, as will generally be the case at a loaded derived frequency). Of course, still other power quantities may be equal to zero, namely, if the loads at these frequencies are either short circuits, open circuits or purely-reactive circuits.

However, which power quantities are positive, which are negative and which are zero cannot be deduced from relations (20) in their general form. Such a deduction would require a complete specification of the entire system, A and B. Only in very simple cases, where only a few frequencies are involved, the average power at any other frequency having been made equal to zero, do the energy relations permit us to deduce the sign of the remaining power quantities. In the simplest cases it is even possible to find the ratio of these quantities without completely specifying the system. Some of these simple cases will be dealt with in the next section.

In many cases power flows into the system at the frequencies generated by an energy source so that P_{n_j} is positive, whereas at all derived frequencies the power $P_{n_1, \ldots, n_k}$ is extracted and hence is negative. But this is not necessarily so. In expressions (20) the basis frequencies ω_j are assumed to be positive. Depending upon the relative magnitude of the various basis frequencies ω_j and the parameters n_j, some of the derived frequencies are then negative and thus reverse the sign of the corresponding power quantities in (20). From this we may immediately deduce that the power associated with a basis frequency may become negative. This happens, for instance, when we take the extreme case that no power can be exchanged at all positive derived frequencies due to selective suppression filters incorporated into the external sub-system. In other words, it may happen that power is delivered by the frequency converting sub-system A to the external sub-system B at a basis frequency. To use yet another formulation: sub-system A may exhibit a negative input resistance at a basis frequency which can partly compensate for the losses in the basis-frequency circuit. As soon as the negative resistance prevails over the positive resistances effective at this particular frequency, sustained oscillations may result.

Where the foregoing happens to be the case we have *parametric power amplification by means of a negative-resistance effect.* Of course the average power associated with a basis frequency cannot be negative for all basis frequencies at the same time. At least one of them has to be positive. The power needed for the amplification is drawn from those other basis-frequency generators that deliver positive average power through a frequency-conversion process.

From the discussion of the expression (22) about the manner in which the power delivered by an energy source is distributed over the loads at the various derived frequencies, we can immediately deduce that there exists yet another principle of amplification. Since the various basis-frequency sources all contribute to the power developed at a derived frequency (the ratio between the contributions being governed by the frequency ratio) the absolute value of the total average power developed at a derived frequency may be larger than that flowing into the system at one of the corresponding basis frequencies. When this is the case we have *parametric power amplification associated with the frequency-conversion process* between that particular basis frequency and that particular derived frequency. Here again the power needed for the amplification is supplied by the other basis-frequency generators.

6. **Some very simple examples.**

As an illustration of the various properties of parametric systems mentioned previously, some very simple examples in which only three or four frequency components are involved will be briefly discussed.

6.1. *Modulators with three frequency components.*

Consider a parametric system connected to an external system which comprises two basis-frequency generators delivering signals with a frequency $\omega_1 = \omega_{1,0}$ and a pump (or local-oscillator) frequency $\omega_2 = \omega_{0,1}$, respectively. Load impedances are assumed to be present at these frequencies and at only one of the derived frequencies, namely at $\omega_{\alpha,\beta} = \alpha\omega_1 + \beta\omega_2$, to be specified later on. Hence, average power is exchanged only at the three frequencies characterized by

$$\left.\begin{matrix} n_1 = 1 \\ n_2 = 0 \end{matrix}\right\}, \quad \left.\begin{matrix} n_1 = 0 \\ n_2 = 1 \end{matrix}\right\} \text{ and } \left.\begin{matrix} n_1 = \alpha \\ n_2 = \beta \end{matrix}\right\}.$$

For this case the relations (20) reduce to:

$$\begin{aligned} \frac{P_{\omega_1}}{\omega_1} + \frac{\alpha P_{\alpha\omega_1+\beta\omega_2}}{\alpha\,\omega_1 + \beta\,\omega_2} &= 0, \\ \frac{P_{\omega_2}}{\omega_2} + \frac{\beta P_{\alpha\omega_1+\beta\omega_2}}{\alpha\,\omega_1 + \beta\,\omega_2} &= 0, \end{aligned} \tag{24}$$

if we write P_{ω_1} for $P_{1,0}$, and P_{ω_2} for $P_{0,1}$.

Since the two equations (24) contain three unknown quantities we can only determine the ratio of these quantities unless we further specify the system, which we will not do. Depending upon the relative magnitudes of ω_1 and ω_2, and upon the values chosen for α and β, we can discriminate between the following cases:

case (*i*): $\quad 0 < \omega_1 < \omega_2,\ \alpha = \beta = 1$.

The relations (24) now assume the form:

$$\begin{aligned} \frac{P_{\omega_1}}{\omega_1} + \frac{P_{\omega_1+\omega_2}}{\omega_1 + \omega_2} &= 0, \\ \frac{P_{\omega_2}}{\omega_2} + \frac{P_{\omega_1+\omega_2}}{\omega_1 + \omega_2} &= 0. \end{aligned} \tag{25}$$

Obviously, since ω_1 and ω_2 are positive quantities, the sign of $P_{\omega_1+\omega_2}$ is opposite to that of P_{ω_1} and P_{ω_2}, which are both positive since at least one of the basis-frequency generators must deliver positive average power. Hence, $P_{\omega_1+\omega_2}$ is negative indicating that the average power is absorbed by the load at the sum frequency $\omega_1 + \omega_2$.

The frequency-conversion process $\omega_1 \rightarrow \omega_1 + \omega_2$ is accompanied by a power gain

$$G = P_{\omega_1+\omega_2} \,/\, P_{\omega_1} = (\omega_1 + \omega_2) \,/\, \omega_1 > 1,$$

as follows directly from the first of the equations (25). This principle of obtaining conversion gain was disclosed by Alexanderson[21]) as far back as 1916. Figure 2i illustrates how P_{ω_1} and P_{ω_2} contribute to $P_{\omega_1+\omega_2}$ depending upon the relative magnitude of ω_1 and ω_2.

This case corresponds to the *upper-sideband modulator*; a signal-frequency band $0 \to \omega_1$ is translated to a band $\omega_2 \to \omega_2 + \omega_1$ without altering the relative position of the individual signal-frequency components. Therefore, it is sometimes[6]) designated as a *non-inverting* modulator.

case (ii): $\quad 0 < \omega_1 < \omega_2\,,\ \alpha = -\beta = 1\,.$

Relations (24) now become:

$$\frac{P_{\omega_1}}{\omega_1} + \frac{P_{\omega_1-\omega_2}}{\omega_1-\omega_2} = 0\,,$$
$$\frac{P_{\omega_2}}{\omega_2} - \frac{P_{\omega_1-\omega_2}}{\omega_1-\omega_2} = 0\,. \tag{26}$$

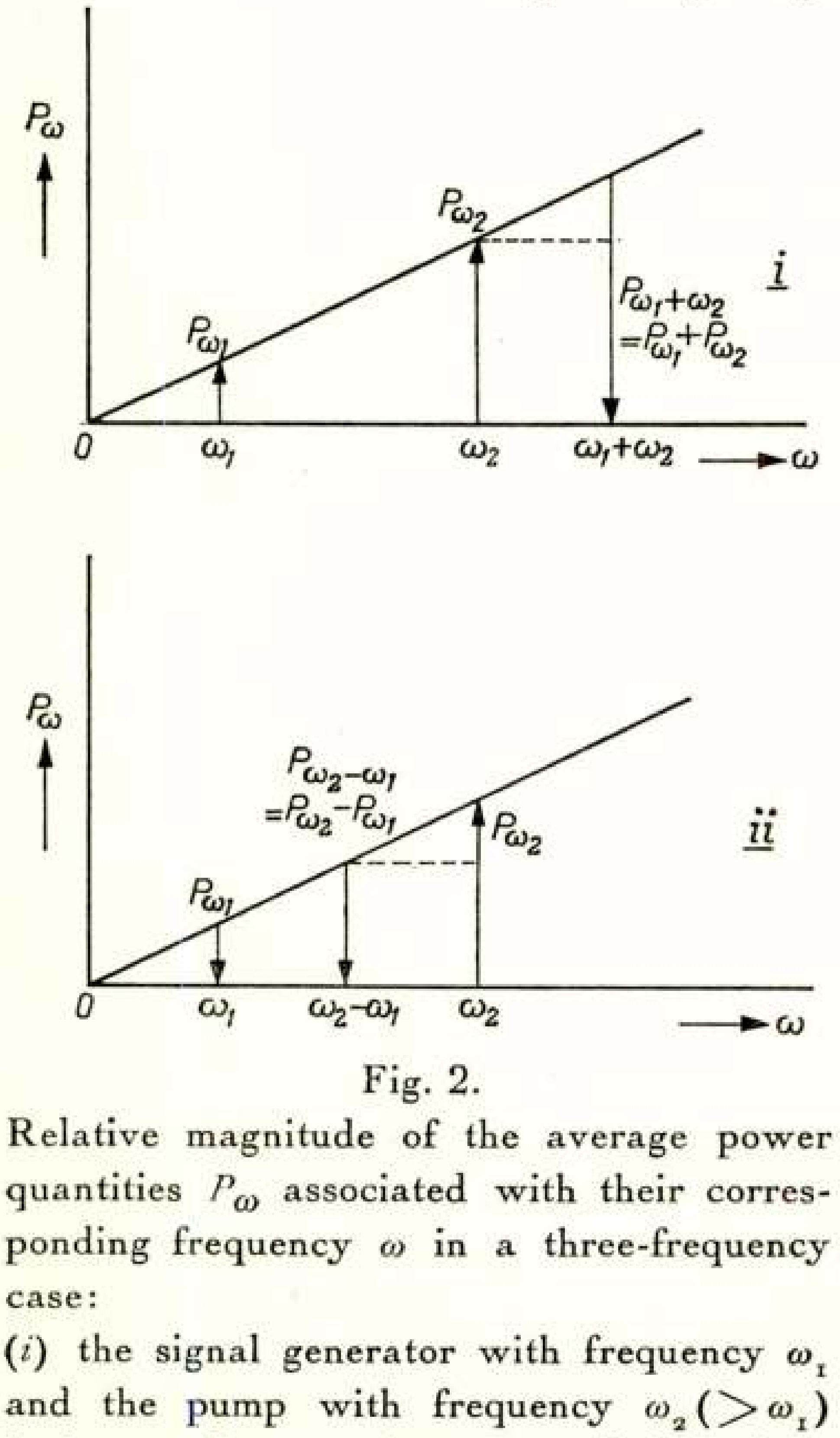

Fig. 2.

Relative magnitude of the average power quantities P_ω associated with their corresponding frequency ω in a three-frequency case:

(*i*) the signal generator with frequency ω_1 and the pump with frequency $\omega_2\,(>\omega_1)$ both contribute to the power dissipated at the sum-frequency $\omega_1+\omega_2$;

(*ii*) the pump with frequency $\omega_2\,(>\omega_1)$ delivers power at both the signal frequency ω_1 and the difference frequency $\omega_2-\omega_1$.

It is clear that now P_{ω_1} and P_{ω_2} have opposite signs and, since $\omega_1 - \omega_2$ is a negative quantity, $P_{\omega_1-\omega_2}$ and P_{ω_1} have the same sign which is negative since power is absorbed at $\omega_1 - \omega_2$. Hence at signal frequency ω_1 average power, instead of being supplied by the signal generator, is delivered to it. This effect may be interpreted as the creation of a negative input resistance at ω_1 which can compensate for the losses present in the signal circuit. This effect, that can be utilized for amplification by means of a negative input-impedance repeater, was discovered by Hartley (cf. ref.[22])) and suggested for practical use by Burton[23]), Boardman[24]) and Peterson[25]). The power needed for the absorption at ω_1 and at $\omega_2 - \omega_1$ is supplied by the pump at ω_2 (see fig. 2ii). The amount of power absorbed at ω_1 depends on the load at $\omega_2 - \omega_1$, which is sometimes called the *idling* frequency.

If $2\omega_1 = \omega_2$, the frequencies ω_1 and $\omega_2 - \omega_1$ have the same value. Depending upon the load at ω_1 sustained oscillations at ω_1 may result, even if at the frequency ω_1 the generator is removed or short-circuited. The power needed for these *second sub-harmonic oscillations* is of course derived from the pump at ω_2 [22]). The application of parametric sub-harmonic oscillators in the computer field, as information-storing elements and as logic devices, has been suggested almost simultaneously, but independently, by Goto [26]), who introduced the name parametron and Von Neumann [27]). In these proposals, two sub-harmonic signals mutually shifted in phase over half a period are introduced to represent the binaries "0" and "1".

If the property of frequency conversion $\omega_1 \to \omega_2 - \omega_1$ is used, rather than the property of presenting a negative input resistance at ω_1, this *lower-sideband modulator* is sometimes referred to as an *inverting* modulator to express the fact that the relative position of the frequencies in the original signal band is reversed after conversion. If this system is used as a modulator, overlapping frequency bands occur so soon as $2\omega_1 > \omega_2$.

case (iii): $\quad 0 < \omega_2 < \omega_1\,,\ \alpha = \beta = 1\,.$

This is in fact the same as case (*i*) if we interchange the role of ω_1 and ω_2. For the frequency conversion of signal bands this case is not very interesting owing to the overlapping of frequency bands.

case (iv): $\quad 0 < \omega_2 < \omega_1\,,\ \alpha = -\beta = 1\,.$

This is a modification of case (*ii*); at ω_1 the signal generator has to deliver power to the load at $\omega_1 - \omega_2$ as well as to the local oscillator at ω_2 which therefore experiences a negative input resistance. Here, too, overlapping of frequency bands occurs. We also see from the first of the equations (26) that *parametric demodulation* results in a *power loss*.

6.2. *Modulators with four frequency components.*

We shall now consider some cases in which four frequencies are involved in order to illustrate some aspects of parametric devices not encountered in the three-frequency cases. We shall take two cases in which the signals at two derived or idling frequencies (viz. $\alpha_1\omega_1 + \beta_1\omega_2$ and $\alpha_2\omega_1 + \beta_2\omega_2$) instead of one are loaded, and one case in which two pump frequencies (ω_2 and ω_3) are involved together with one idling frequency.

case (i): $\quad 0 < \omega_1 < \omega_2, \ \alpha_1 = \alpha_2 = -\beta_1 = \beta_2 = 1.$

For this case relations (20) reduce to:

$$\begin{aligned} \frac{P_{\omega_1}}{\omega_1} + \frac{P_{\omega_1-\omega_2}}{\omega_1-\omega_2} + \frac{P_{\omega_1+\omega_2}}{\omega_1+\omega_2} &= 0, \\ \frac{P_{\omega_2}}{\omega_2} - \frac{P_{\omega_1-\omega_2}}{\omega_1-\omega_2} + \frac{P_{\omega_1+\omega_2}}{\omega_1+\omega_2} &= 0. \end{aligned} \tag{27}$$

Since we have now four unknown power quantities and only two equations we are unable to determine even the ratio between these quantities. Hence no expression can be derived for the gain unless the entire system, the loads inclusive, is specified. However, one peculiarity can be deduced from eqs (27) without reference to a specified system.

We have seen in the foregoing sub-section under cases (*i*) and (*ii*) that the loading at the sum-frequency $\omega_1 + \omega_2$ creates a positive input resistance at the signal frequency ω_1, whereas the loading at the difference frequency $\omega_2 - \omega_1$ gives rise to a negative input resistance at ω_1. Since the magnitudes of $P_{\omega_1-\omega_2}$ and $P_{\omega_1+\omega_2}$ depend upon the loads, which can be arbitrarily chosen, it must be possible to fulfil the condition:

$$\frac{P_{\omega_1-\omega_2}}{\omega_1-\omega_2} + \frac{P_{\omega_1+\omega_2}}{\omega_1+\omega_2} = 0,$$

which implies

$$P_{\omega_1} = 0.$$

In other words, by a proper loading at the derived frequencies it is possible in the case of parametric *double-sideband modulators* to eliminate the reaction of the parametric system upon the signal generator as was demonstrated by Tellegen and Duinker [28]). In this case we can state that the conversion gain is infinite.

case (ii): $0 < \omega_2 < \omega_1 < 2\,\omega_2,\ \alpha_1 = \alpha_2 = -\,\beta_1 = -\,\beta_2 / 2 = 1\,.$

In the foregoing we have seen that a negative input resistance may be obtained if the parametric system is used as an up-converter, i.e., if $\omega_1 < \omega_2$. However, in principle it is possible to obtain a negative input resistance, and hence under favourable conditions sustained oscillations, at a frequency ω_1 higher than the pump frequency as was first suggested by Hogan, Jepsen and Vartanian [29]).

For the chosen parameters the relations (20) assume the form:

$$\frac{P_{\omega_1}}{\omega_1} + \frac{P_{\omega_1-\omega_2}}{\omega_1-\omega_2} + \frac{P_{\omega_1-2\omega_2}}{\omega_1-2\,\omega_2} = 0,$$
$$\frac{P_{\omega_2}}{\omega_2} - \frac{P_{\omega_1-\omega_2}}{\omega_1-\omega_2} - 2\,\frac{P_{\omega_1-2\omega_2}}{\omega_1-2\,\omega_2} = 0. \tag{28}$$

Since $\omega_1 - \omega_2$ is positive, and $\omega_1 - 2\,\omega_2$ is negative, whereas $P_{\omega_1-\omega_2}$ and $P_{\omega_1-2\omega_2}$ are both negative quantities owing to the loads at their corresponding frequencies, eqs (28) are obeyed if P_{ω_1} is a negative quantity, i.e. if:

$$\frac{P_{\omega_1-\omega_2}}{\omega_1-\omega_2} + \frac{P_{\omega_1-2\omega_2}}{\omega_1-2\,\omega_2} > 0.$$

This condition can be satisfied by a proper choice of the loading impedances at the two derived (or idling) frequencies.

case (iii): $0 < \omega_2 < \omega_3 < \omega_1,\ \alpha = -\,\beta_1 = -\,\beta_2 = 1\,.$

This case which was indicated by Bloom and Chang [30]) presents another possibility for obtaining a negative input resistance at a signal frequency higher than the pump frequencies ω_2 and ω_3. Relations (20) now become:

$$\frac{P_{\omega_1}}{\omega_1} + \frac{P_{\omega_1-\omega_2-\omega_3}}{\omega_1-\omega_2-\omega_3} = 0,$$
$$\frac{P_{\omega_2}}{\omega_2} - \frac{P_{\omega_1-\omega_2-\omega_3}}{\omega_1-\omega_2-\omega_3} = 0, \tag{29}$$
$$\frac{P_{\omega_3}}{\omega_3} - \frac{P_{\omega_1-\omega_2-\omega_3}}{\omega_1-\omega_2-\omega_3} = 0.$$

If ω_1, ω_2 and ω_3 are all positive and $\omega_1 - \omega_2 - \omega_3$ is negative while power is delivered by the pumps (i.e., P_{ω_2} and P_{ω_3} are both positive) but extracted at the idling frequency so that

$P_{\omega_1 - \omega_2 - \omega_3}$ is negative, then P_{ω_1} is negative, which is the desired result.

If ω_2 and ω_3 are equal, the last two of the eqs (29) are identical, whereas the idling frequency becomes $\omega_1 - 2\,\omega_2$ implying that the second harmonic of the pump frequency must be present in the signal generated by the pump. In this case the negative input-resistance at ω_1 is obtained via a higher harmonic of the pump frequency.

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RADIO TELECOMMUNICATIE CONFERENTIES GENEVE 1959

Van 17 augustus tot 17 december van dit jaar vindt te Genève een internationale Conferentie betreffende telecommunicatie plaats. Deze „Union Internationale des Télécommunications" conferentie staat onder leiding van C. J. Acton (Canada) en er wordt aan deelgenomen door omstreeks 600 afgevaardigden van 83 van de uit 101 leden bestaande U.I.T.

De U.I.T. is de oudste inter-gouvernementale organisatie (opgericht in 1865) en is één van de gespecialiseerde organen van de Verenigde Naties. Onder meer behoort ook tot een van haar taken de internationale registratie van de frequenties.

De thans plaatsvindende conferentie heeft als voornaamste doel de herziening van de „Radio Regulations" van Atlantic City 1947. Meer dan 5000 wijzigingsvoorstellen zijn door de diverse landen ingediend. Deze worden in hoofdzaak behandeld in 4 commissies, namelijk de commissies 4, 5, 6 en 7.

Commissie 4: Herziening van de frequentie indeling.

Commissie 5: Internationale frequentie registratie en Internationale frequentielijst.

Commissie 6: Technische vraagstukken.

Commissie 7: Deze commissie behandelt de algemene procedure betrekking hebbende op het radio verkeer van mobile stations.

Aangezien het onmogelijk is dat de vele voorstellen in deze hierboven genoemde uiteraard grote groepen behandeld worden zijn er een groot aantal (± 60) sub-commissies en andere groepen gevormd, die zich met speciale vraagstukken bezighouden.

Door een aantal landen wordt getracht om te voorkomen dat er in de indeling van het frequentie-spectrum, zoals momenteel opgenomen in de Atlantic City Regulations, wijzigingen worden aangebracht. Wel zal de frequentie-tabel naar de hogere frequenties worden uitgebreid tot 40000 Mc/s. (De huidige frequentietabel loopt tot 10500 Mc/s.) Nieuw zullen de frequentie banden zijn, waarover gediscussieerd wordt, ten behoeve van de radio-communicatie met satellieten en planeten. Bovendien verlangt de Radio-astronomie bescherming van een aantal frequentie-banden.

Hoewel de grote problemen praktisch onoplosbaar zijn, in het bijzonder door de bekende politieke tegenstellingen, zal het resultaat wel zijn, dat in veel gevallen een compromis wordt bereikt, dan wel dat een regeling wordt verkregen, welke door zijn algemeenheid vele wegen openlaat.

Op 15 oktober a.s. is bovendien de z.g. plenipotentiaire Conferentie van de U.I.T. begonnen.

Deze houdt zich bezig met de meer structurele vraagstukken van de U.I.T. Bovendien zullen de toegenomen activiteiten van de U.I.T. behandeld worden en het aan deze organisatie ten grondslag liggende verdrag. Dit verdrag is voor de laatste maal in 1952 te Buenos Aires herzien.

Van Nederlandse zijde wordt aan eerstgenoemde Conferentie deelgenomen door:

Namens de P.T.T.: Prof. Ir. G. H. Bast. Dir. Generaal.
Ir. J. D. H. van der Toorn, Oud Dir. Gen.
Ir. A. J. Ehnle. Hfd. directeur.
G. M. Brinkman. Adj. Inspecteur.
A. C. Fortgens. Adj. Inspecteur.
P. de Groen. Oud Inspecteur.
C. van Geel. Directeur I.
J. Houtsmuller. Hfd. ing. bijz. dienst.
Ir. B. J. Stöver. Hfd. Ing. Alg. dienst.
Ir. J. C. Verton. Hfd. Ing. Alg. dienst.
P. E. Willems. Referendaris.
Jhr. Dr. Ir. G. Th. F. van der Wijck. Hfd. Ing. Alg. dienst.

Namens Rijks Luchtvaartdienst:

O. I. Selis. Directeur afd. Luchtverkeerbeveiliging.
A. A. de Roode. Hfd. Inspecteur.
H. Dené. Hfd. commies-A.

Namens het ministerie van Defensie:

J. H. R. van der Willigen. Luitenant ter Zee 1e kl.

Namens Radio Nederland Wereld-omroep:

E. van Eldik. Plv. hfd. v. d. Tech. dienst.

Namens de K.L.M.:

J. F. Varekamp. Hfd. bureau Operationele Verbindingen.
Z. J. van de Hoek Ostende.

Namens Radio-Holland N.V.:

Ir. H. T. Hylkema. Hfd. Ing.

SYMPOSIUM I.R.E.

Door het Institute of Radio Engineers zal op 11, 12 en 13 januari 1960 gehouden worden het 6th National Symposium on Reliability and Quality Control in Electronics.

Plaats: Hotel Statler, Washington D.C.

Belangstellenden kunnen zich voor nadere inlichtingen wenden tot de Secretaris of tot Ir. J. C. Diels, techn. wetensch. attaché, Nederlandse Ambassade, 1470 Euclid Street, Washington 9, D.C., U.S.A.

FIRST MEETING OF THE BENELUX SECTION OF THE INSTITUTE OF RADIO ENGINEERS

On October 3rd the newly formed Benelux Section of the IRE held their first meeting. This took place at the Rotterdam Offices of Radio-Holland N.V.

Benelux Section of the I.R.E. At the speaker's table, from left to right: Ir. C. B. Broersma, Chairman of Membership Comittee; Dr. H. P. Williams, Chairman of Publicity Committee; Mrs. B. D. H. Tellegen; Prof. B. D. H. Tellegen, Fellow I.R.E. (and N.R.G.); Dr. L. V. Berkner, Fellow I.R.E. and Director I.R.E.; Ir. H. Rinia, Chairman of Benelux Section.

where some 75 members and guests heard talks on the radio and navigational equipment of the new s.s. Rotterdam by Messrs. Ir. C. B. Broersma and Ir. H. T. Hylkema of Radio-Holland and A. Wepster of the Holland America Line. The lectures were preceded by an opening address from Chairman Ir. H. Rinia. During the lunch which followed, Dr. L. V. Berkner, President of U.R.S.I. and a Director of the IRE, presented the good wishes of the Board of Directors and emphasized the international character of the Institute and its interest in promoting contact among radio engineers from all lands.

In the afternoon the whole party went aboard the new H.A.L. flagship "Rotterdam" and were shown the radio station and also the navigational equipment on the bridge. Since many other parts of this fascinating ship were also visited, the whole occasion was quite a memorable one. Even the weather contributed to this by being exceptionally warm and sunny.

BOEKBESPREKING

Principles of Transistor Circuits: Introduction to the Design of Amplifiers, Receivers and other Circuits. By S. W. Amos, B.Sc. (Hons.), A.M.I.E.E. Published for "Wireless World" on 26 March 1959 by Iliffe & Sons Limited. Formaat 8¾" x 5½". 167 blz., 105 figuren in de tekst. Prijs 21 s.

Dit boekje is uitgegeven onder auspiciën van de Wireless World. In tien hoofdstukken wordt op een voor ieder begrijpelijke wijze een eenvoudige handleiding gegeven voor het ontwerpen van transistor-schakelingen. Het is zodanig geschreven, dat na negen hoofdstukken te hebben doorgewerkt, een radioamateur zonder hulp een eenvoudige superheterodyne radio-ontvanger met transistoren kan ontwerpen en bouwen. In het laatste hoofdstuk wordt een multivibrator gegeven, terwijl enkele nieuwe typen hf transistoren worden besproken.

De schrijver is op de praktijk ingesteld; hij gaat niet diep op de stof in. Over de oorzaak van het afwijkende gedrag van transistoren wordt weinig gezegd. Wel wijdt hij terecht een — uitstekend — hoofdstuk aan de stabilisatie van het instelpunt.

In de eerste vijf hoofdstukken worden ook puntcontact-transistoren behandeld. Dit werkt verwarrend, daar hier negatieve uit- en ingangsimpedanties kunnen voorkomen, terwijl dit type transistor toch niet meer in gebruik is. De schrijver had dit beter kunnen laten vervallen.

Dit boekje zal voor de radio-amateur en de middelbaar radio-technicus van veel nut zijn.

E. E. P. P.

Fluctuation Phenomena in Semiconductors, door A. van der Ziel Uitgegeven door Butterworth Scientific Publications, London 1959. 168 bladz., prijs 35 sh.

Het doel van dit boek is een overzicht te geven van de stand van zaken in het theoretisch en experimenteel onderzoek van fluctuatie-verschijnselen in half-geleider materialen, en de daarvan afgeleide producten, zoals photogeleiders, thermistors, diodes, transistoren.

De ruis-eigenschappen van vierpolen en de meting van de karakteristieke parameters worden evenals de gebruikelijke wiskundige methoden kort gerecapituleerd. Generatie-recombinatie ruis (veroorzaakt door ontstaan en verloren gaan van ladingsdragers, holen en gaten) wordt uitvoerig behandeld. Voor flickereffect wordt Mc Whorter's theorie als meest belovend gevolgd. De theoretische discussie van ruis bij photogeleiders is gebaseerd op het werk van van Vliet. Ruis in halfgeleider weerstanden en thermistors wordt kort behandeld. Uitvoerige hoofdstukken zijn gewijd aan shot-ruis in diodes en transistoren. Deze zijn voor een groot deel gebaseerd op het werk van de schrijver en zijn collega's aan de University of Minnesota. Er volgt dan een discussie van flicker ruis in diodes en transistoren (oppervlakte- en lekstroom effecten). Het slot hoofdstuk behandelt toepassingen en beschouwingen over minimaal ruisgetal. Hierbij wordt kort ingegaan op de ruiseigenschappen van parametrische versterkers, van het type diodes met variabele capaciteit.

De nadruk ligt in dit boek op een uitvoerige wiskundige discussie van het physisch model. Gemeten krommen zoekt men er vergeefs in, al wordt wel steeds naar de experimentele literatuur verwezen. Wat de theorie betreft vindt men er echter alles bijeen, wat in de laatste jaren (tot 1959!) aan belangrijke ideeën voortgebracht is, critisch bezien vanuit één gezichtspunt. Het boek is helder geschreven, maar door de aard van het onderwerp geen gemakkelijke lectuur. De uitgave is keurig verzorgd. Een belangrijke bijdrage tot de halfgeleider literatuur!

F. L. S.

bezorgden. Op 24 augustus werd hij benoemd tot buitengewoon hoogleraar te Leiden om onderwijs te geven in de microgolftechniek en de toepassing ervan in de sterrekunde, natuurkunde en scheikunde. Prof. Muller is beheerder van de Radiosterrenwacht te Dwingeloo en zal deze functie naast zijn hoogleraarsambt blijven vervullen. Hij werkt thans aan ruis- en stabiliteitsproblemen bij hoge frequenties en aan een aantal astronomische waarnemingsprogramma's. De laatste tijd heeft hij zich ook bezig gehouden met masers en parametrische versterkers voor radioastronomische doeleinden.

VOORGESTELD LID

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