Numerical modelling as defined in Encyclopædia Britannica:

"A computer-generated description of a mathematical system to represent the behaviour of a real or proposed system that uses a set of equations and inequalities to represent the functional relationships within the system."

Numerical modelling of ice & ice-structure interactions

KIVI NIRIA

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34e lustrum 170 jaar TU Delft

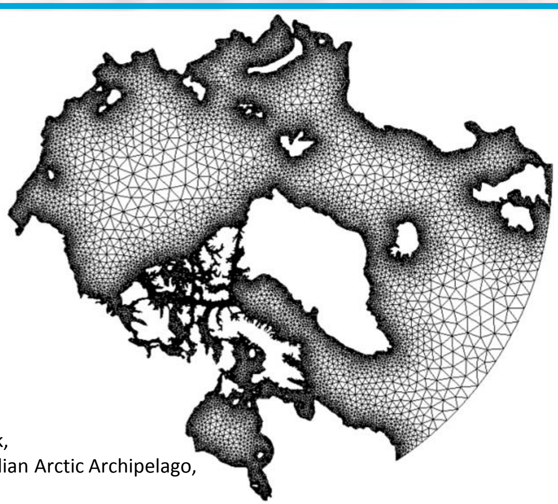
Arctic Battle

Symposium - 8 March 2012

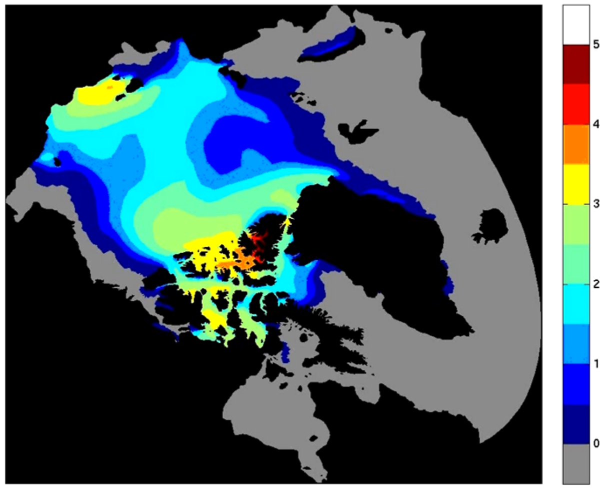
Current uses for numerical modelling

Numerical modelling is widely used for applications in Arctic Engineering:

- Flow of ice sheets
- Thermodynamic growth & decay of ice
- Response of sea ice to climate variations
- Ice concentrations
- Oceanography



Terwisscha van Scheltinga, Myers & Pietrzak,
A finite element sea ice model of the Canadian Arctic Archipelago,
Ocean Dynamics 60, 1539-1558, 2010



Modeled mean ice thickness for September 1979-2009

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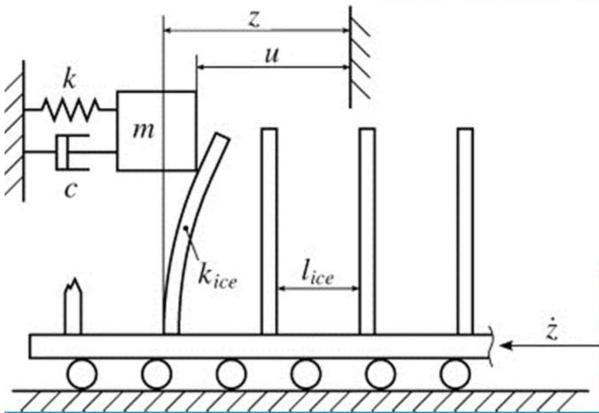
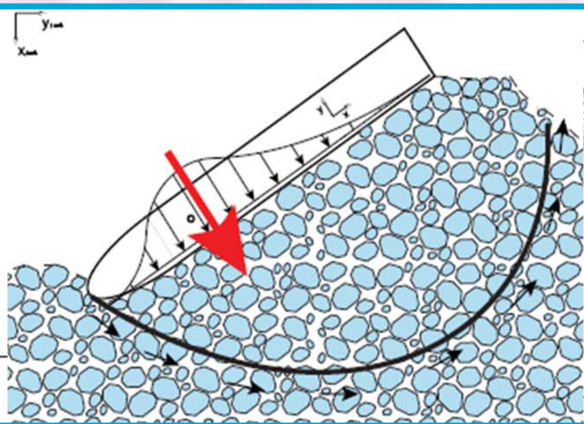
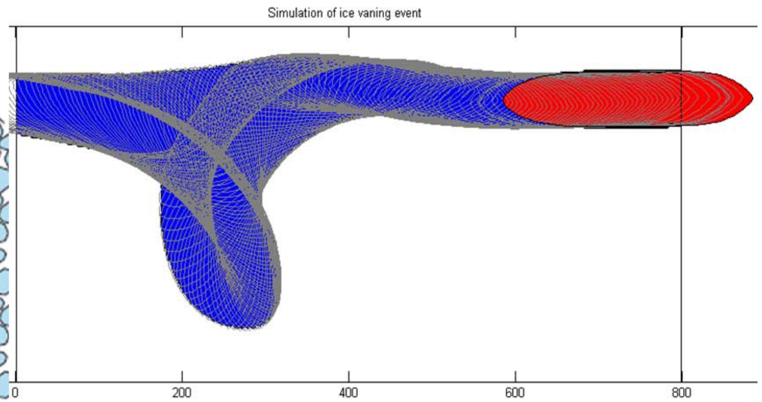
Numerical applications for ice-structure interaction

Current and past numerical modelling for ice-structure interaction:

- Phenomenological modelling of dynamic ice-structure interaction, by: Kärnä - VTT Finland / Huang & Liu - CAS China
- Ice-structure interaction between ships and broken ice, by: Sayed, Frederking & Barker - NRC Canada / Løset - NTNU Norway
- 'Simplistic' ice-structure interaction models based on static ice loads from ISO19906 combined with Ansys, by: Hidding & Bonnafoux, MSc. DUT/SBM







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Our purpose for numerical modelling

We are however interested in:

- The behaviour of ice when its loaded
- The way ice fails against different types of structures, and thus
- The loads that ice sheets or ice floes exert on offshore structures

Our focus is the interaction of offshore structures with first-year level ice, because:

- Loads due to level ice floes are norm-ative over those due to broken ice fields
- It is the most common ice feature at locations considered for development

Thus, for now, neglecting the critical ice loads due to for example ice ridges or icebergs

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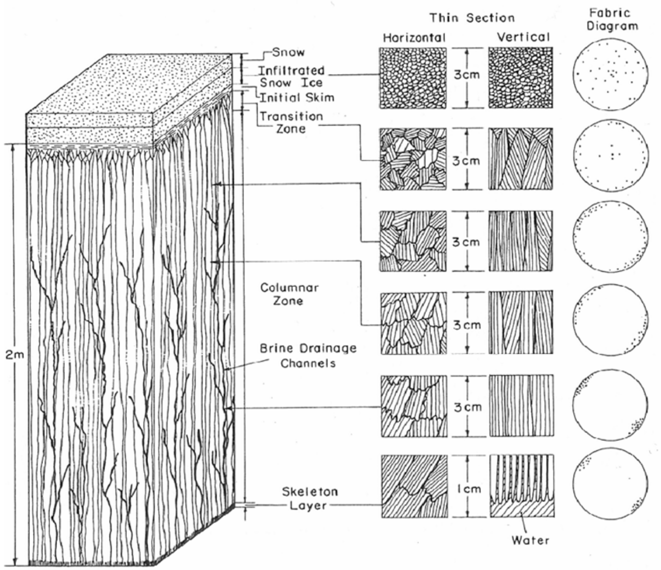
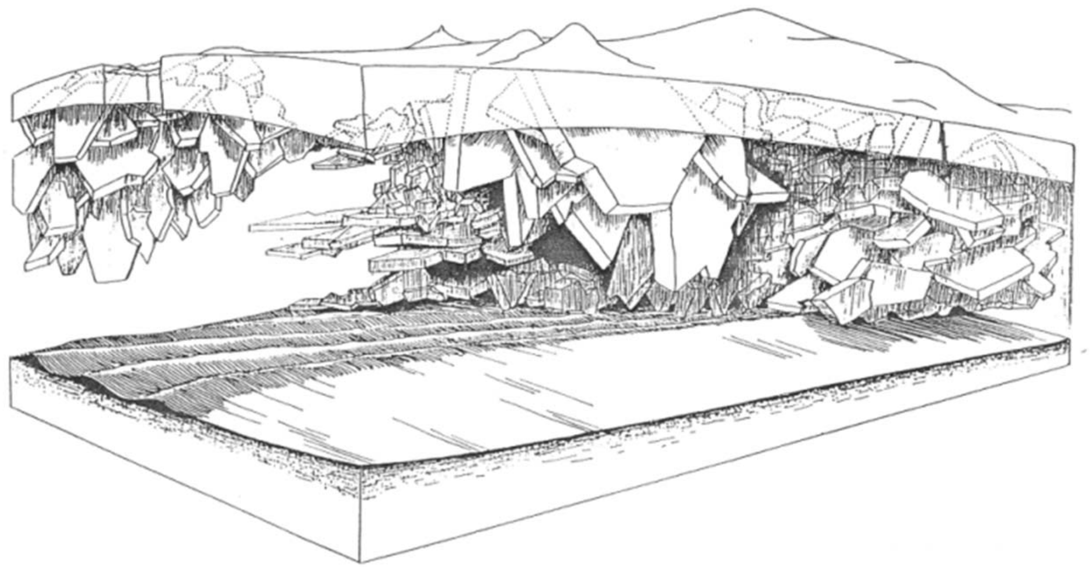
Typical properties of ice

- Ice is a heterogeneous natural material
- Ice is anisotropic
- Ice has a variable shape that is unknown and therefore uncertain
- Ice-structure interactions are large scale ⟶ scaled model tests





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Model-scale versus full-scale tests

At first, from scaled model tests:

"Failure of sea ice was thought to be well-described by plastic limit analysis"

(Plastic limit analysis = Linear elasticity until the critical stress is reached, then yield until total failure)

But when some full-scale data became available, it was found that:

"The measured forces exerted by moving ice on an oil platform may be an order of magnitude smaller than the predictions based on laboratory tests!"

This is due to the size effect on structural strength:

- Statistical size effect
- Energetic (or deterministic) size effect

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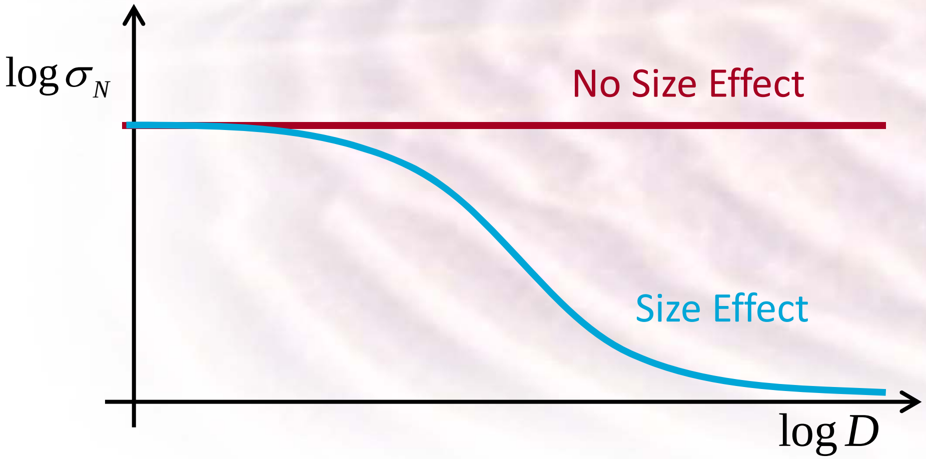
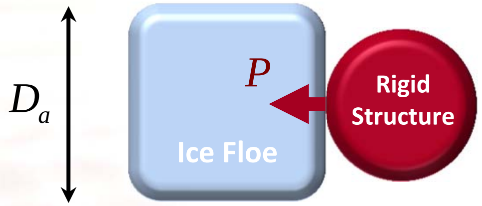
The principle of size effect

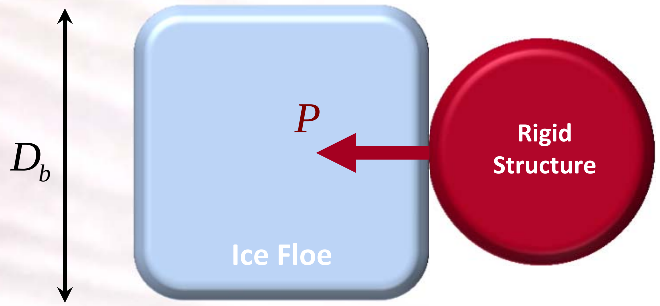
Nominal strength: $\sigma_N = C_\sigma \dfrac{P}{D^2}$

No Size Effect: $$\frac{\sigma_N^a}{\sigma_N^b} = \frac{f\left(D_a^2\right)}{f\left(D_b^2\right)} = f\left(\frac{D_a^2}{D_b^2}\right)$$

Size Effect: $$\frac{\sigma_N^a}{\sigma_N^b} = \frac{f\left(D_a^2\right)}{f\left(D_b^2\right)} \neq f\left(\frac{D_a^2}{D_b^2}\right)$$







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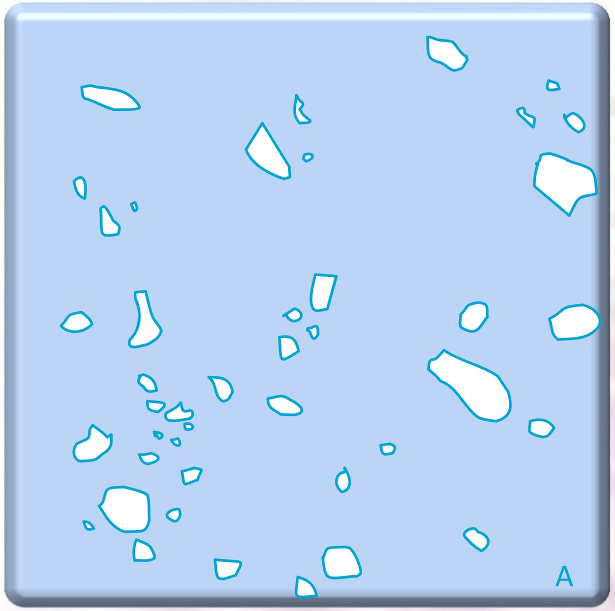
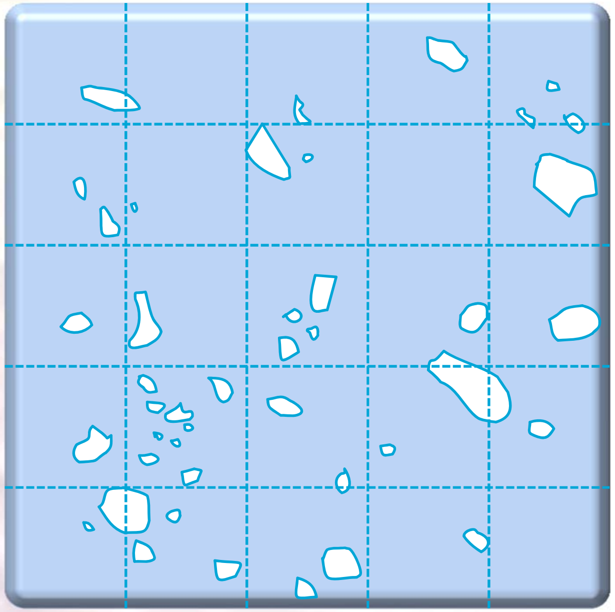
Statistical size effect





The average of the nominal strength of all specimen in floe A is the same as the nominal strength of floe A

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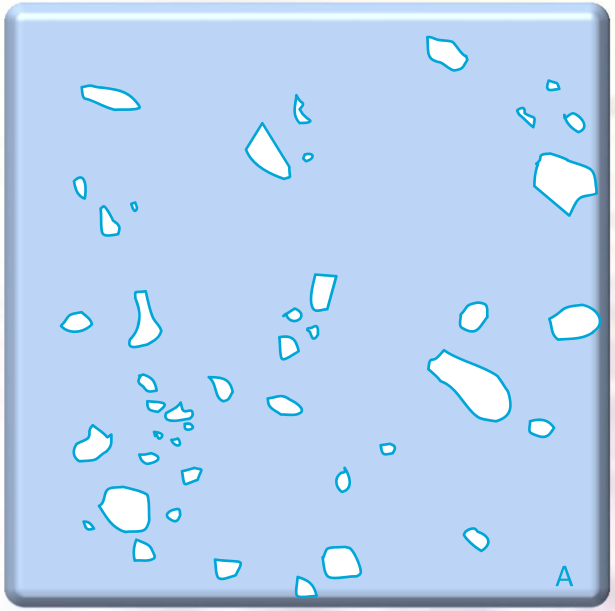
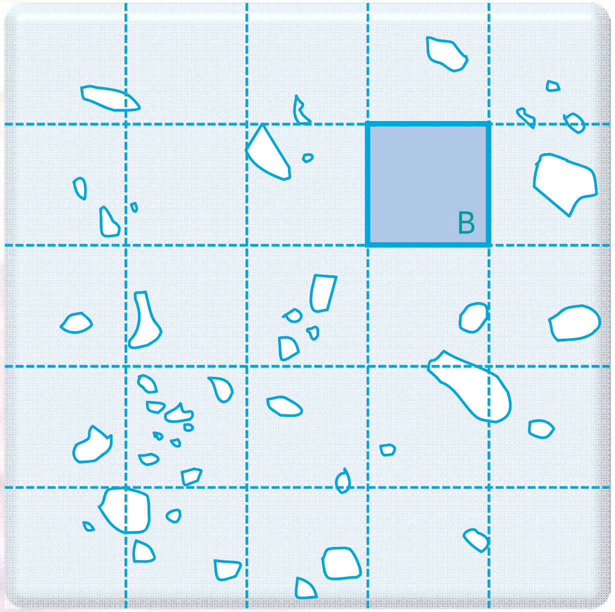
Statistical size effect

Scale B up to the size of A





Floe B is much stronger then floe A: $\sigma_N^A < \sigma_N^B$

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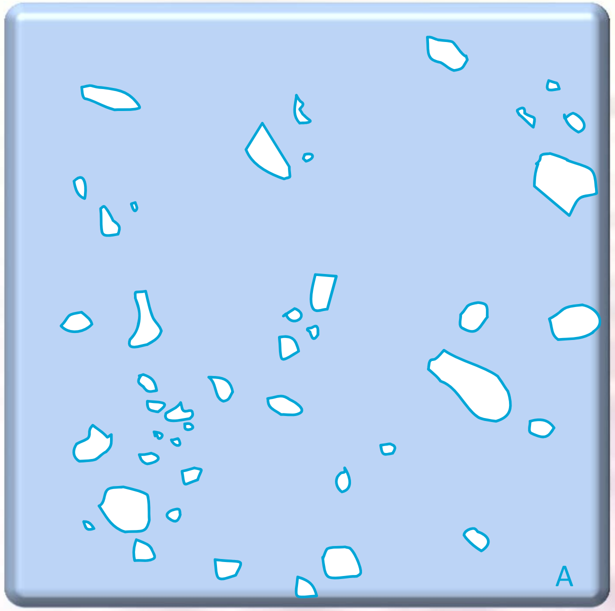
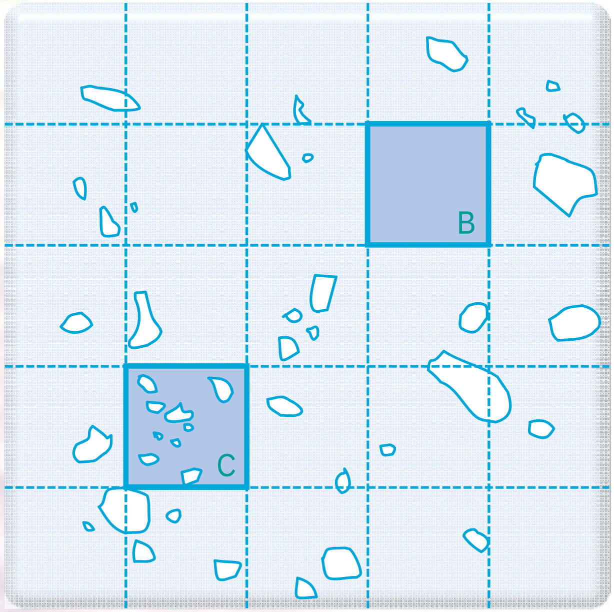
Statistical size effect

Ice floe A will not fail at specimen B, but for example at specimen C!





- An ice floe is as strong as its weakest link
- The bigger the floe, the higher the chance on a relatively weak spot

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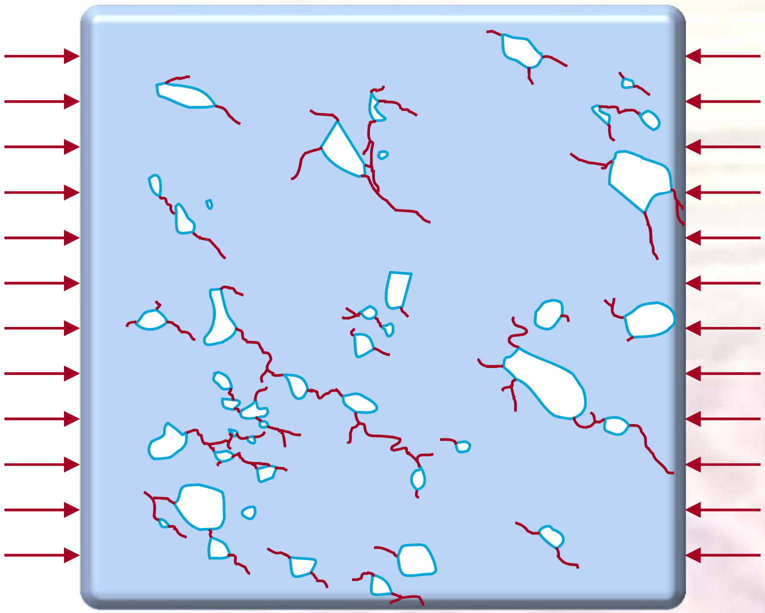
Energetic size effect



- When loaded, tiny cracks appear at weak spots in the ice
- Under continuous loading, these cracks propagate due to peak stresses at the crack tip and stress relief in the surrounding ice, i.e. stress redistribution
- The strength of an ice floe then depends on the energy dissipation due to crack propagation, which is independent of the floe size.

In large ice floes, crack propagation occurs at relatively lower stresses then crack propagation in small ice floes.

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Influence size effect on ice-failure

So generally, we can state that:

"The nominal strength of ice decreases relative to the floe size"

Due to the size effect, ice can be considered a quasi-brittle material:

- Small ice floes interacting with small structures:
  Macro-failure in a more ductile manner due to distributed micro-cracking
- Large ice floes interacting with large structures:
  Macro-failure in an almost perfect brittle manner

Also the relative velocity between an ice floe and a structure influences the way an ice floe fails during ice-structure interaction:

- Low velocities ⟶ ductile
- High velocities ⟶ brittle

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Preferred practice due to the size effect

1. Create a numerical model that:
   - *captures the desired phenomena*, and
   - *includes the size effect*

2. Perform scale-model tests

3. Tune the parameters of the numerical model to the scale-model tests

4. Use the resulting numerical model to obtain results for full-scale situations

This is known as: "*Numerical Upscaling*"

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Numerical approaches

The choice of the model and thus the numerical approach to be used, mainly depends on the scale of the phenomena one wishes to capture

The different scales are:

- Macro → Global phenomena
- Meso → Local phenomena
- Micro → Microstructure phenomena
- Nano → Molecular phenomena

Macro-scale modelling for global phenomena:

- Continuum models can sometimes be (partially) solved analytically
- Numerically solving these models requires relatively short calculation time
- Continuum models do not capture all properties of an ice floe

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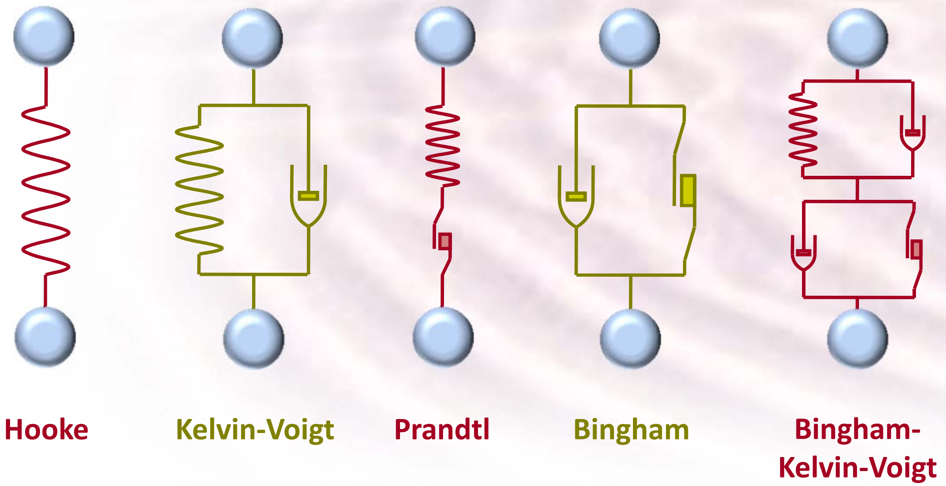
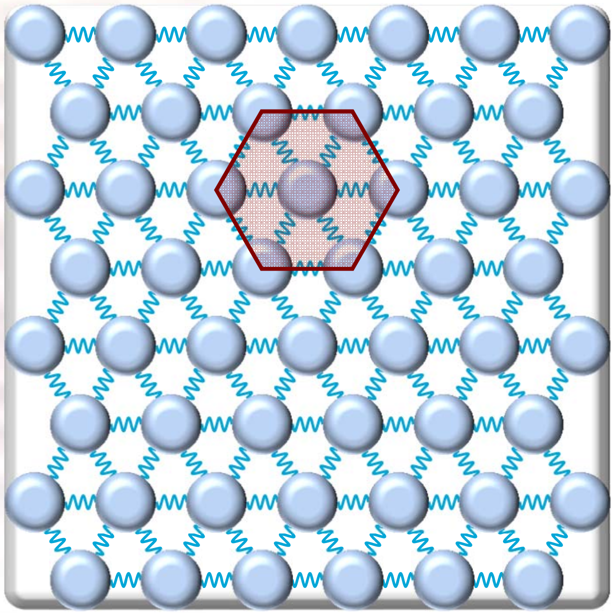
Meso-scale modelling: the hexagonal lattice

- The ice floe is modelled as a hexagonal system of rigid particles
- Other geometrical configurations of the particles can be made

- The interaction between any 2 adjacent particles is described by kinematic elements that may have various properties:





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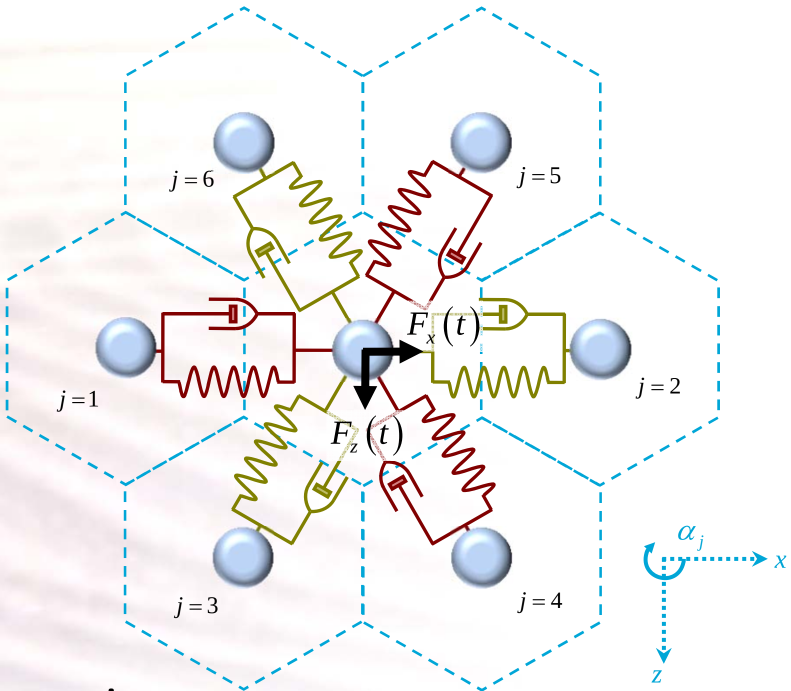
A hexagonal lattice with Kelvin-Voigt elements

- At every node in the lattice, we have 2 equations of motion, respectively describing the equilibria in x- and z-direction:

$$M\ddot{u}_x - \hat{K}_e \sum_{j=1}^{6} e_j \cos\alpha_j = F_x(t)$$

$$M\ddot{u}_z - \hat{K}_e \sum_{j=1}^{6} e_j \sin\alpha_j = F_z(t)$$

$$\underline{\underline{M}}\,\underline{\ddot{u}} + \underline{\underline{C}}\,\underline{\dot{u}} + \underline{\underline{K}}\,\underline{u} = \underline{F}(t)$$



- The properties of the particles and the kinematic elements may be varied, enabling inhomogeneity

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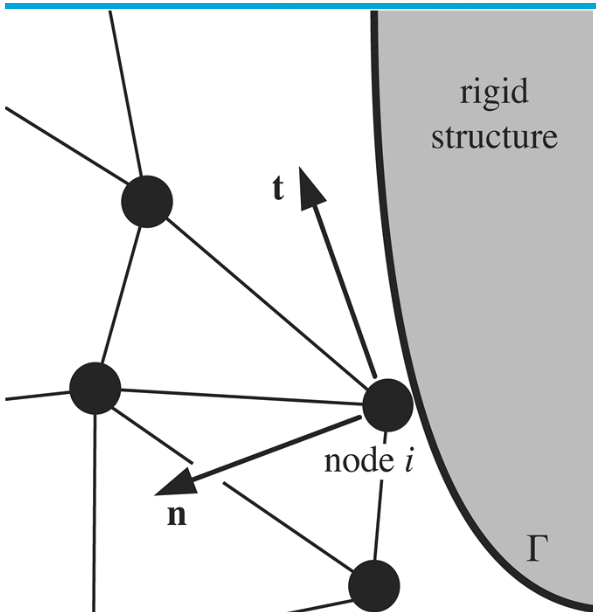
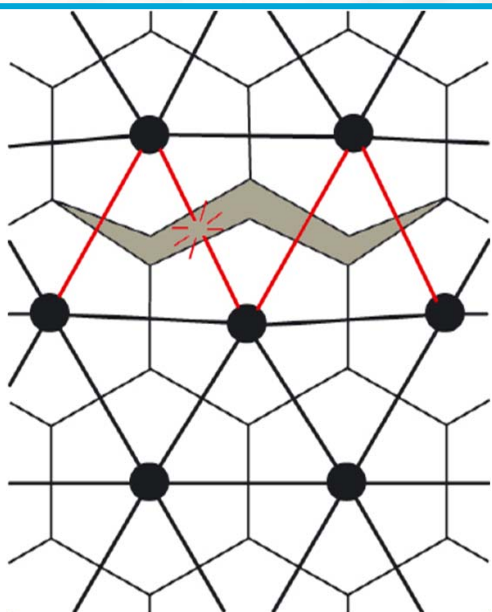
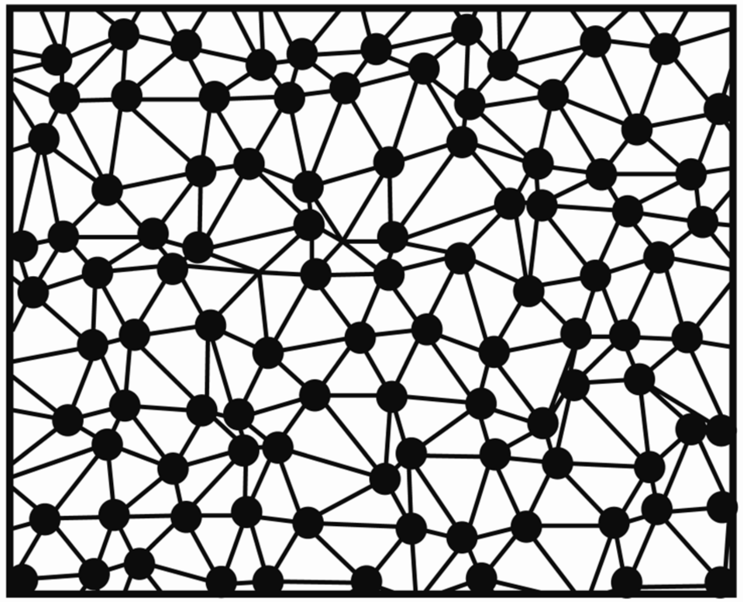
Crack propagation in the lattice

- When the force in an element between 2 adjacent nodes becomes too large, a crack appears and that element is deleted
- Using a regular mesh, crack propagation is mesh dependent, therefore the mesh is randomized







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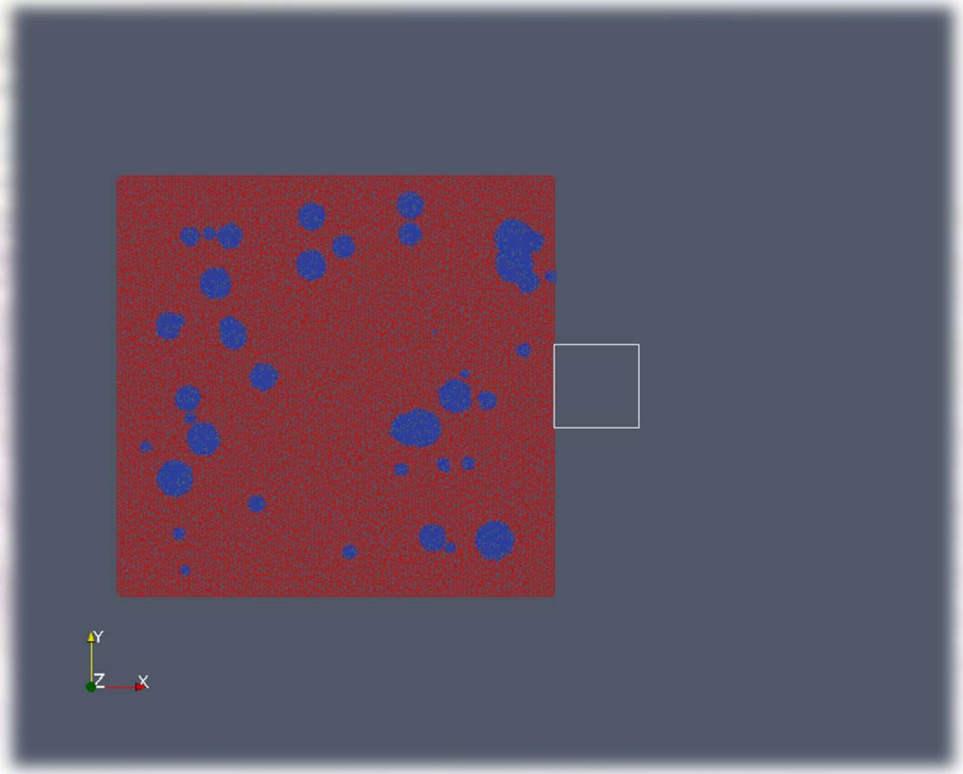
Ice crushing against a rigid structure

- A lattice model to simulate ice crushing against a rigid structure, by: Dr. O. Dorival and Prof. A. Metrikine (2008)



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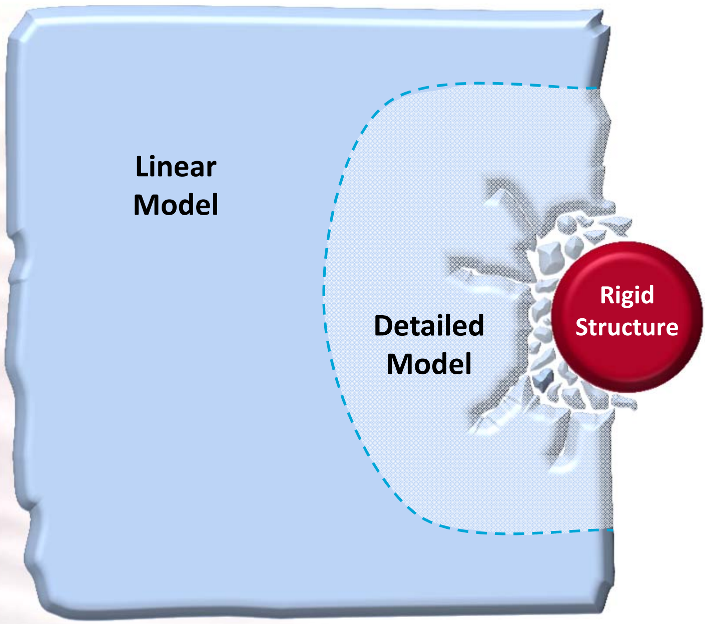
Perspective: A coupled discrete-continuous model

With the aim to improve usability and to limit calculation time, we divide the floe into 2 segments:

- The part of the ice floe close to the ice-structure interaction is modelled with high detail using the discrete lattice to account for the nonlinear behaviour of the ice
- At a distance from the ice-structure interaction, the behaviour of the ice floe is linear and thus modelled as a linear continuum
- We then couple the discrete lattice and the continuum such that the wave reflection at the lattice-continuum interface is minimal



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A non-reflective lattice-continuum interface

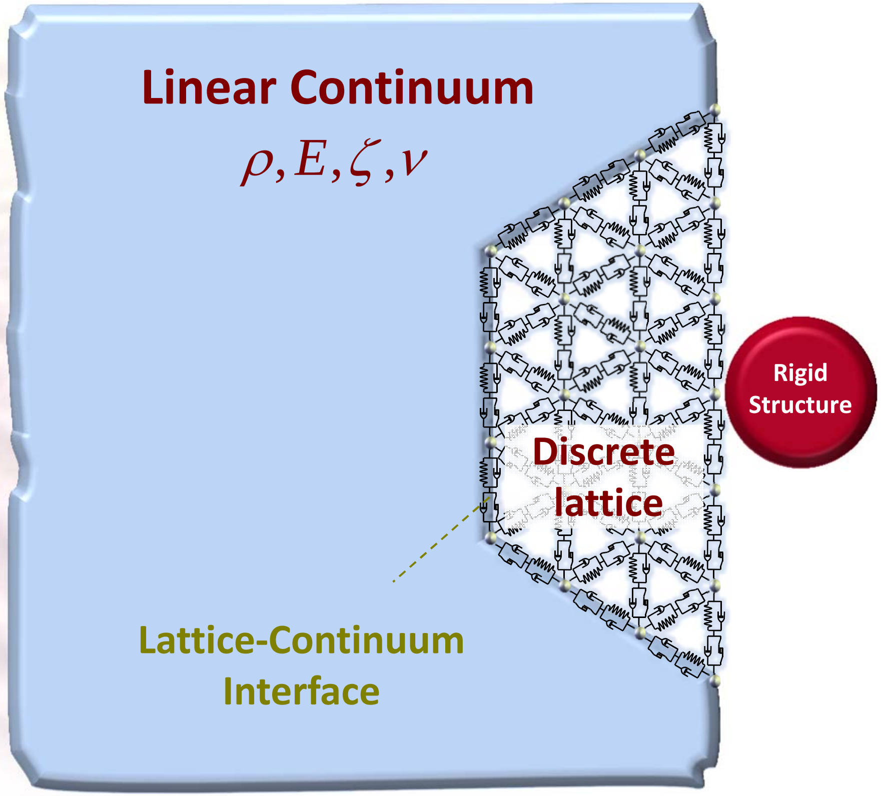
Dynamic stiffness is *"The ratio between a dynamic load and its response"*

The dynamic stiffness is described by the force-displacement relation in the frequency domain as:

$$F(\omega) = \chi(\omega) u(\omega)$$

The force-displacement relation in the time domain is then found applying the inverse Laplace transform, and is found as the convolution of $\chi(t)$ and $u(t)$:

$$F(t) = \int_0^t \chi(t-\tau) u(\tau) d\tau$$



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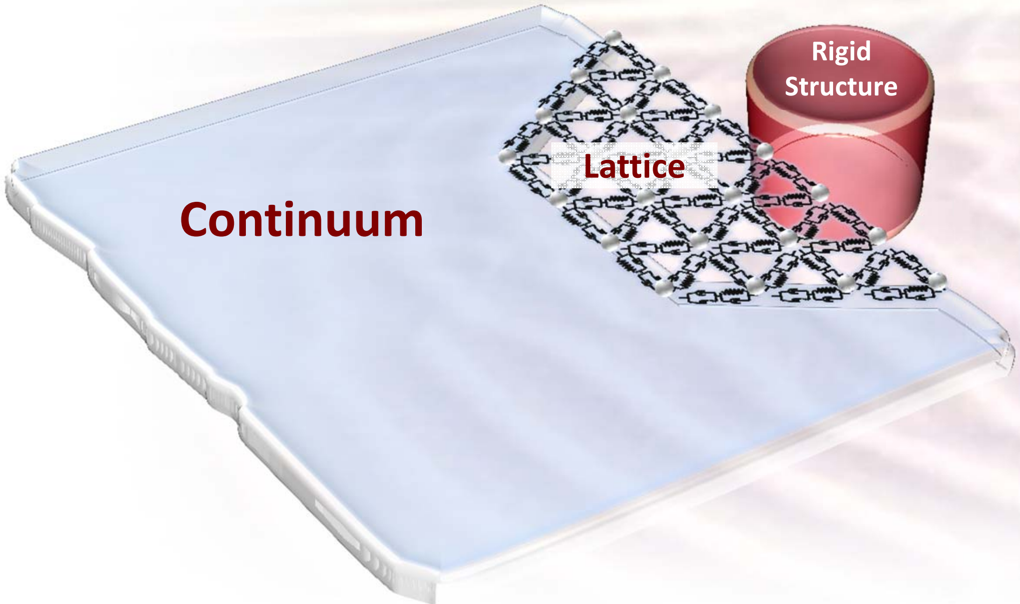
Perspective: Extension into 3D

Extending the model into 3D by adding the vertical dimension allows for:

- Clearance of rubble under the ice sheet
- The option to model the bending of an ice floe against a conical structure



Hydralab IV – HyIII-HSVA-04 – Kärnä et al.

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Numerical modelling of ice-structure interaction

- The models that can be studied analytically often do not capture the phenomena that are observed in reality during ice-structure interaction
- Scale model tests give insight in the phenomena of ice-structure interaction but can not be scaled straightforwardly to actual size problems
- To obtain a better description and understanding of ice and ice-structure interaction, it is a necessity to model these phenomena numerically
- Additionally, numerical models are preferred over model- and full-scale tests for cost effective design

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Thank you!

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